Math 161 - Calculus - Exam 1 - Guide **Name:** ____________________

February 2, 2024

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1. Evaluate the following limits.:

(1.1) $\lim_{x \to 4} \frac{1}{x}$

(1.2) $\lim_{x \to 5} 3$

(1.3) $\lim_{x \to 4} \frac{1}{x-4}$

(1.4) $\lim_{x \to 5} \frac{-1}{(x-5)^2}$

(1.5) $\lim_{x \to 7} \frac{1}{x-4}$

(1.6) $\lim_{x \to +\infty} \frac{1}{x-4}$

(1.7) $\lim_{x \to +\infty} \frac{\cos(2x)}{x}$

(1.8) $\lim_{x \to 0} \frac{\sin(x)}{x}$

(1.9) $\lim_{x \to +\infty} e^x \cos(x)$

(1.10) $\lim_{x \to 4} \left[\frac{2}{x-4} - \frac{2}{x^2-7x+12}\right]$

(1.11) $\lim_{x \to +\infty} x - \sqrt{x^2-6x}$

2. (Give a short written response) What does the derivative tell you about a function?

3. Use the definition of the derivative to compute $f'(x)$.

(3.1) $f(x) = \dfrac{3}{x-1}$

(3.2) $f(x) = 5\sqrt{x+2}$

4. Find all solutions

(4.1) $3e^x + 5 = e^x + 11$

(4.2) $\left(1 + \frac{0.06}{12}\right)^{2x} = 4$

(4.3) $\frac{50}{1 + 2e^{3x}} = 10$

5. Find $f^{-1}(x)$.

(5.1) $f(x) = \frac{1 - 4x}{3x + 2}$

neg. leading coef & degree 2 ⇒ ∩

6. Given

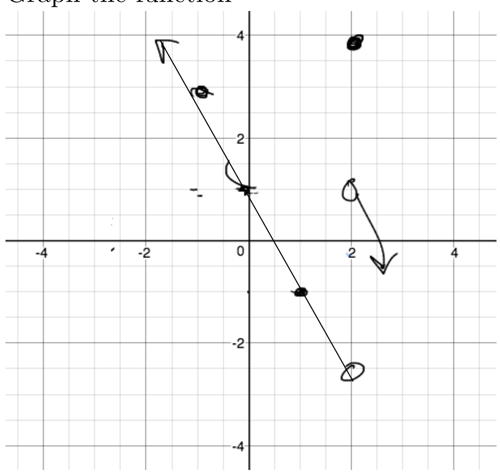
$$f(x) = \begin{cases} 5 - x^2 & x > 2 \\ -2x + 1 & x < 2 \\ 4 & x = 2 \end{cases}$$

line w/ slope −2, y-int = 1

point (2, 4)

(6.1) Graph the function

only graph on the section determined by inequality.



$\lim_{x \to 2^-} f(x) = -3$

plug in $x = 2$ into

$-2x + 1$

$-2(2) + 1$

$= -3$

(6.2) Finish the definition below:
A function $f(x)$ is continuous at $x = a$ if

$$\lim_{x \to a} f(x) = f(a)$$

$\lim_{x \to 2^+} f(x) = 1$

plug into right piece

$5 - (2)^2$

(6.3) Use the definition of continuity to show that $f(x)$ is not continuous at $x = 2$.

$$\lim_{x \to 2} f(x) = \text{DNE} \neq f(2)$$

$\lim_{x \to 2^-} f(x) = -3$ $\quad \lim_{x \to 2^+} f(x) = 1$

(6.4) Is there a way to define $f(x)$ at $x = 2$ so that $f(x)$ is continuous at $x = 2$? Why or why not?

what if f(2) were another number. could it be continuous then?

no. the limit needs to exist first, i.e., the two sides of the curve have to agree at x = 2

5. According to Peak Oil Theory, first proposed in 1956, the total amount of crude oil $Q(t)$ produced worldwide up to time $t$ has a graph like the one shown below.

$\frac{2.3-2.28}{2110-2090} = \frac{.02}{20}$

.001 $\frac{tb}{yr}$

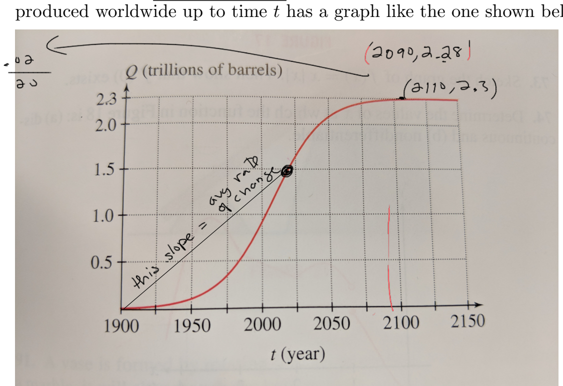


FIGURE 14 Total oil production up to time $t$.

6.1

$\frac{Q(2020) - Q(1900)}{2020 - 1900}$

$= \frac{1.5 - 0}{120} = .0125$ $\frac{\text{trillion barrels}}{\text{year}}$

1,000,000,000,000

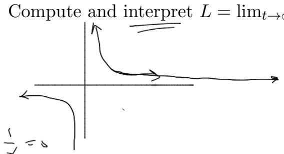
$= 12.5$ $\frac{\text{billion barrels}}{\text{year}}$

6.2 $\approx 0$ or $.001$ $\frac{tb}{yr}$

(6.1) Estimate the average rate of change (slope of secant line) of oil production from 1900 to 2020.

(6.2) Estimate the instanenous rate of change of oil production at the year 2100. (slope of tangent line)

(6.3) Compute and interpret $L = \lim_{t\to\infty} Q(t)$. $= 2.3$ trillion barrels



$\lim_{x\to\infty} \frac{1}{x} = 0$

seconds is given by $y = 58t - .83t^2$.

(a) Find the average velocity over the given time intervals:

(7.1) time interval: [1,1.5] ________________

(7.2) time interval: [1,1.01] ________________

(7.3) time interval: [1,1.001] ________________

(7.4) Find the instantaneous velocity after one second (to the nearest hundredth).

8. The position of a cat running from a dog down a dark alley is given by the values of the table.

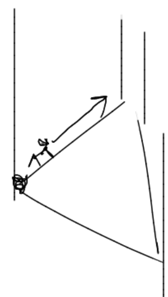
| t (seconds) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| s (feet) | 0 | 14 | 30 | 73 | 100 | 117 |

← s(t)

Find the average velocity of the cat for the time period beginning with $t = 2$ and lasting

Avg. Rate of Change of position over a time interval

(8.1) 3 seconds  Interval: [2,5]  $\frac{s(5)-s(2)}{5-2} = \frac{117-30}{3} = \frac{87}{3} = 29 \frac{ft}{sec}$

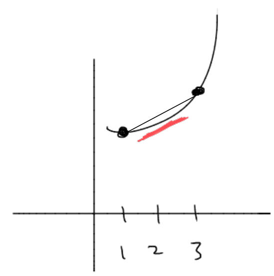


(8.2) 2 seconds  [2,4]

(8.3) 1 seconds [2,3]

Estimate the instantaneous velocity when $t = 2$ by finding the average velocity from $t = 1$ to $t = 3$.

$[1,3] \rightsquigarrow \frac{73-14}{3-1} = \frac{59}{2} = 29.5 \frac{ft}{sec}$




Do you think this is a good estimate or not? Explain.

This is a good estimate because in the previous window the ARoC was 16, and in the following window it was 43. The average of these two is 29.5

This is a bad estimate because in the first two windows the ARoC's were 14 - 16, but in the one after 43. The cat likely sped up AFTER second two, thus the instantaneous rate of change at 2 is likely close to 15 - 20.

Another good question

A cat's distance from a dog at time t is given by s(t) = t^2 + 1. feet , t = sec

Find the Avg. Velocity of the cat over the interval [1,3]. $\frac{s(3) - s(1)}{3 - 1} = \frac{10 - 2}{2} = 4 \text{ ft/sec}$

Find the instantaneous velocity of the cat at time t = 2.

derivative: power rule

$s'(t) = 2t$

plug in $t = 2$ $s'(2) = 4$