Math 161 - Calculus - Exam 1 - Guide **Name:** ______________________
September 12, 2024
On the exam you must show your work to receive full credit.

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1. Evaluate the following limits.:

(1.1) $\lim_{x\to 4} \frac{1}{x}$

(1.2) $\lim_{x\to 5} 3$

(1.3) $\lim_{x\to 4} \frac{1}{x-4}$

(1.4) $\lim_{x\to 5} \frac{-1}{(x-5)^2}$

(1.5) $\lim_{x\to 0} \frac{\sqrt{x^3+13}\sin(x)}{x}$

(1.6) $\lim_{x\to +\infty} \frac{1}{x-4}$

(1.7) $\lim_{x\to +\infty} \frac{\cos(2x)}{x}$

(1.8) $\lim_{x\to 0} \frac{\sin(x)}{x}$

(1.9) $\lim_{x\to +\infty} e^x \cos(x)$

(1.10) $\lim_{x\to 4} \left[\frac{2}{x-4} - \frac{2}{x^2-7x+12}\right]$

(1.11) $\lim_{x\to 0} \frac{\sin(7x)}{\sin(5x)}$

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(1.5) $\lim_{x \to 0} \frac{\sqrt{x^3+13}\sin(x)}{x}$

(1.6) $\lim_{x \to +\infty} \frac{1}{x-4} = 0$ b/c $\frac{1}{\text{large}+} \approx 0$

(1.7) $\lim_{x \to +\infty} \frac{\cos(2x)}{x}$ $\frac{\sim\sim\sim}{\text{huge}}$ $\frac{-1}{x} \le \frac{\cos(2x)}{x} \le \frac{1}{x}$ $\frac{-1}{x} \to 0$, $\frac{1}{x} \to 0$

so Squeeze Thm $\Rightarrow$ $\frac{\cos(2x)}{x} \to 0$

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2. (Give a short written response) What does the derivative tell you about a function?

- Instantaneous rate of change
- Slope of the tangent line to the graph.
- when the derivative is positive, the function is increasing
  (negative) (decreasing)

3. Use the definition of the derivative to compute $f'(x)$.

(3.1) $f(x) = \dfrac{3}{x-1}$

(3.2) $f(x) = 5\sqrt{x+2}$

4. For what values of A and B is $f(x)$ continuous?

$$f(x) = \begin{cases} 6x^{-1} & x < -1 \\ Ax + B & -1 \leq x \leq \frac{1}{2} \\ 8x^{-1} & x > \frac{1}{2} \end{cases}$$

agree on $x = -1$

agree on $x = 1/2$

use top

$\lim_{x \to -1} f(x) = f(-1)$ use middle $= A(-1)+B$

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+}$

① $6(-1)^{-1} = A(-1) + B = -A + B$

$-6 = \frac{6}{-1}$

$\boxed{-6 = -A + B}$

② $A\left(\frac{1}{2}\right) + B = 8\left(\frac{1}{2}\right)^{-1} = 16$

$\boxed{\frac{A}{2} + B = 16}$ $\xrightarrow{\text{times } 2}$ $A + 2B = 32$

$A = 32 - 2B$

sub into #1

$-6 = -(32-2B) + B$

$= -32 + 2B + B$

$= -32 + 3B$

$32 - 6 = 3B$

$26 = 3B$

$\frac{26}{3} = B$

③ combine boxes:

- Isolate B in box #1

$A - 6 = B$

- sub this ↑ into box #2

$\frac{A}{2} + A - 6 = 16$

- Isolate A $\frac{3}{2}A = 22$

$\boxed{A = \frac{44}{3}}$

④ Plug this A back in box #1:

$-6 = -\left(\frac{44}{3}\right) + B$

$\frac{26}{3} = \frac{44}{3} - \frac{\overset{18}{6 \cdot 3}}{3} = B$

$\boxed{B = \frac{26}{3}}$

5. For functions, what is the relationship between the concepts: derivative, increasing and decreasing?

6. Find all solutions

$\ln(3e^x+5) \neq x \cdot \ln(3 \cdot e + 5)$

(6.1) $3e^x + 5 = e^x + 11$

$2e^x = 3e^x - e^x = 6 \dashrightarrow e^x = \frac{6}{2} = 3$

$\ln(2e^x) = \ln 6$

$\ln 2 + \underbrace{\ln e^x}_{x} = \ln 6$

$\boxed{x = \ln 6 - \ln(2)}$

or

$\ln(e^x) = \ln(3)$

$\boxed{x = \ln(3)}$

same

$\ln(A^x) = x \cdot \ln(A)$

(6.2) $\left(\underbrace{1 + \frac{0.06}{12}}_{\#}\right)^{2x} = 4$

$\ln\left(1 + \frac{0.06}{12}\right)^{2x} = \ln 4$

$2x \cdot \ln\left(1 + \frac{0.06}{12}\right) = \ln 4$

$x = \dfrac{\ln 4}{2 \cdot \ln\left(1 + \frac{0.06}{12}\right)}$

stop here!

or

$\left[\left(1 + \frac{0.06}{12}\right)^x\right]^2 = 4$

$\downarrow \sqrt{\ }$ both

$\left(1 + \frac{0.06}{12}\right)^x = 2$

$\downarrow \log_2$

$x \cdot \log_2\left(1 + \frac{0.06}{12}\right) = \log_2 2 = 1$

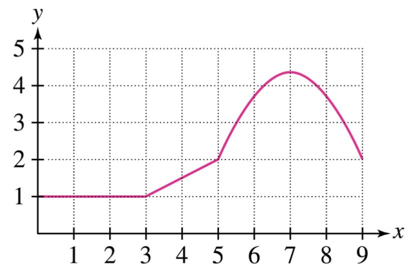
$x = \dfrac{1}{\log_2\left(1 + \frac{0.06}{12}\right)}$

(6.3) $\dfrac{50}{1 + 2e^{3x}} = 10$

7. Find $f^{-1}(x)$.

(7.1) $f(x) = \dfrac{1 - 4x}{3x + 2}$

8. Assume the graph of $f(x)$ is below. For what values of a is $f'(a)$ positive, negative and zero?



9. Suppose that f has a domain of [7, 17] and a range of [2, 17].
   (9.1) What are the domain and range of the function y = f(x) + 4?
   (9.2) What are the domain and range of the function y = f(x + 4)?

10. According to Peak Oil Theory, first proposed in 1956, the total amount of crude oil $Q(t)$ produced worldwide up to time t has a graph like the one shown below.

1,500,000,000,000

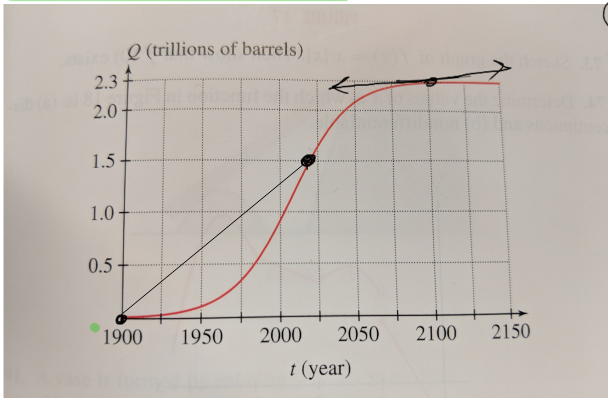


FIGURE 14 Total oil production up to time t.

10.1 $\frac{f(b)-f(a)}{b-a}$

$$\Rightarrow \frac{Q(2020)-Q(1900)}{2020-1900}$$

$$\approx \frac{1.5-0}{120}\ \frac{\text{trillion barrel}}{\text{year}}$$

$$\approx \frac{1{,}50\cancel{0}}{12\cancel{0}}\ \frac{\text{billion barrels}}{\text{year}} \approx 12\ \frac{\text{billion barrels}}{\text{year}}$$

(10.1) Estimate the average rate of change of oil production from 1900 to 2020. (Hint: This is the slope of the corresponding secant line.)

(10.2) Estimate the instanenous rate of change of oil production at the year 2100. (Hint: This is the derivative at the year 2100.) $\approx .1\ \frac{\text{trill barrells}}{\text{year}} \approx 0\ \frac{\text{trillion barrels}}{\text{year}}$

(10.3) Compute and interpret $L = \lim_{t\to\infty} Q(t)$.

$\lim_{t\to\infty} Q(t) = 2.3$ trillion barrels