Math 161 - Calculus - Exam 1 - Guide **Name:** ______________________

September 12, 2024

On the exam you must show your work to receive full credit.

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1. Evaluate the following limits.:

(1.1) $\lim_{x\to 4} \frac{1}{x} = \frac{1}{4}$

(1.2) $\lim_{x\to 5} 3 = 3$

quick sol'n: address degree

odd

(1.3) $\lim_{x\to 4} \frac{1}{x-4} =$ DNE b/c

$\lim_{x\to 4^-} \frac{1}{x-4} \approx \frac{1}{3.999-4} \approx \frac{1}{\text{small negative}} \approx \text{large negative} = -\infty$

$\lim_{x\to 4^+} \frac{1}{x-4} \approx \frac{1}{4.0001-4} \approx \frac{1}{\text{small positive}} \approx \text{large positive} = +\infty$

(1.4) $\lim_{x\to 5} \frac{-1}{(x-5)^2} = -\infty$

even

$\lim_{x\to 5^-} \frac{-1}{(x-5)^2} \approx \frac{-1}{(4.99-5)^2} = \frac{-1}{\text{small } +} = -\infty$

$\lim_{x\to 5^+} \frac{-1}{(x-5)^2} \approx \frac{-1}{(5.001-5)^2} = \frac{-1}{\text{small } +} = -\infty$

(1.5) $\lim_{x\to 0} \frac{\sqrt{x^3+13}\sin(x)}{x}$

(1.6) $\lim_{x\to +\infty} \frac{1}{x-4}$

(1.7) $\lim_{x\to +\infty} \frac{\cos(2x)}{x}$

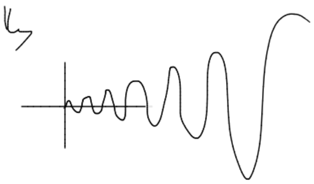
(1.8) $\lim_{x\to 0} \frac{\sin(x)}{x}$

(1.9) $\lim_{x\to +\infty} e^x \cos(x)$

makes amplitude exponentially large

oscillates forever

DNE



(1.10) $\lim_{x\to 4} \left[\frac{2}{x-4} - \frac{2}{x^2-7x+12}\right]$

(1.11) $\lim_{x\to 0} \frac{\sin(7x)}{\sin(5x)}$

2. (Give a short written response) What does the derivative tell you about a function?

3. Use the definition of the derivative to compute $f'(x)$. $\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$

(3.1) $f(x) = \dfrac{3}{x-1}$

$$\lim_{h\to 0} \frac{\frac{3}{x+h-1} - \frac{3}{x-1}}{h} = \lim_{h\to 0} \frac{\overbrace{\frac{3}{x+h-1}\cdot\frac{x-1}{x-1}}^{3x-3} - \overbrace{\frac{3}{x-1}\cdot\frac{x+h-1}{x+h-1}}^{-3x-3h+3}}{h}$$

$-3(x+h-1)$

$$= \lim_{h\to 0} \frac{\frac{-3h}{(x+h-1)(x-1)}}{\cancel{h}} \cdot \frac{1}{h} = \lim_{h\to 0} \frac{-3}{(x+h-1)(x-1)} \overset{\text{d.s.}}{=} \frac{-3}{(x-1)^2}$$

$h = 0$

(3.2) $f(x) = 5\sqrt{x+2}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{5\sqrt{x+h+2} - 5\sqrt{x+2}}{h}$$

$$= 5 \cdot \lim_{h\to 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h} \cdot \frac{\sqrt{x+h+2} + \sqrt{x+2}}{\sqrt{x+h+2} + \sqrt{x+2}}$$

conjugate rad.

$$= 5 \cdot \lim_{h\to 0} \frac{(\cancel{x}+h\cancel{+2}) - (\cancel{x}\cancel{+2})}{h(\sqrt{x+h+2} + \sqrt{x+2})}$$

$$= 5 \cdot \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+2} + \sqrt{x+2})} \overset{\text{d.s.}}{=} 5 \cdot \frac{1}{2\sqrt{x+2}} = \frac{5}{2\sqrt{x+2}}$$

4. For what values of A and B is $f(x)$ continuous?

$$f(x) = \begin{cases} 6x^{-1} & x < -1 \\ Ax + B & -1 \le x \le \frac{1}{2} \\ 8x^{-1} & x > \frac{1}{2} \end{cases}$$

5. For functions, what is the relationship between the concepts: derivative, increasing and decreasing?

If $f'(x) < 0$ then f is decreasing at x.

If $f'(x) > 0$ then f is increasing at x

6. Find all solutions

$\dashrightarrow \ln(A^x) = x \cdot \ln A$ ✓

not $\ln(A^x + B) \neq x \cdot \ln(A+B)$

(6.1) $3e^x + 5 = e^x + 11$

gather e^x terms

$2e^x = 3e^x - e^x = 6$

$\boxed{e^x = \frac{6}{2} = 3}$

$\ln(e^x) = \ln 3$, where $\ln(e^x) = x \cdot \underbrace{\ln(e)}_{=1}$

$x = \ln 3$

(6.2) $\left(1 + \frac{0.06}{12}\right)^{2x} = 4$

$\ln\left(1 + \frac{0.06}{12}\right)^{2x} = \ln(4)$

$2x \cdot \underbrace{\ln\left(1 + \frac{0.06}{12}\right)}_{} = \underbrace{\ln(4)}_{\text{just a \#}}$

$x = \dfrac{\ln(4)}{2 \cdot \ln\left(1 + \frac{0.06}{12}\right)}$

$\left[\left(1 + \frac{0.06}{12}\right)^x\right]^2 = 4$

↓ sqrt both sides

$\left(1 + \frac{0.06}{12}\right)^x = 2$

↓ $\log_2$

$x \cdot \log_2\left(1 + \frac{0.06}{12}\right) = \log_2(2) = 1$

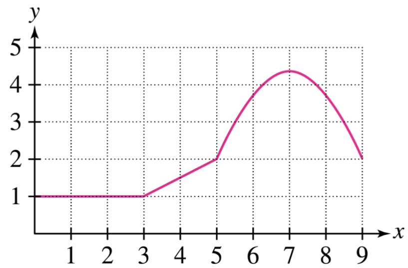
$x = \dfrac{1}{\log_2\left(1 + \frac{0.06}{12}\right)}$

(6.3) $\dfrac{50}{1 + 2e^{3x}} = 10$

7. Find $f^{-1}(x)$.

(7.1) $f(x) = \dfrac{1 - 4x}{3x + 2}$

8. Assume the graph of $f(x)$ is below. For what values of a is $f'(a)$ positive, negative and zero?



9. Suppose that f has a domain of [7, 17] and a range of [2, 17].
   (9.1) What are the domain and range of the function y = f(x) + 4?
   (9.2) What are the domain and range of the function y = f(x + 4)?

10. According to Peak Oil Theory, first proposed in 1956, the total amount of crude oil $Q(t)$ produced worldwide up to time t has a graph like the one shown below.

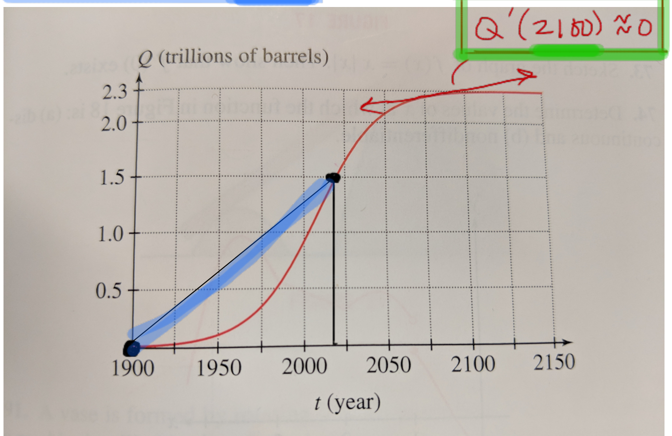


FIGURE 14 Total oil production up to time t.

A.R. of Change: $\dfrac{f(b)-f(a)}{b-a}$

$\dfrac{1.5-0}{2020-1900} = \dfrac{1.5 \text{ tb}}{120}$

$= \dfrac{1{,}500{,}000{,}000{,}000}{120}$ b/y

$= \dfrac{1{,}50\cancel{0}}{12\cancel{0}}$ b.b/y

≈ 12 bb/y

(10.1) Estimate the average rate of change of oil production from 1900 to 2020. (Hint: This is the slope of the corresponding secant line.) est. slope

(10.2) Estimate the instanenous rate of change of oil production at the year 2100. (Hint: This is the derivative at the year 2100.) → slope of the tan. line ≈ 0, in (0,1)

(10.3) Compute and interpret $L = \lim_{t\to\infty} Q(t)$ = 2.3 trillion