Math 161 - Calculus - Exam 1 **Name:** ____________________

February 7, 2024

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1. Evaluate the following limits.:

(1.1) $\lim_{x\to 6} \frac{8}{x-6}$

(1.2) $\lim_{x\to 6} \frac{1}{(x-6)^2}$

(1.3) $\lim_{x\to +\infty} \frac{-1}{x-4}$

(1.4) $\lim_{x\to 0} \frac{\sin(x)}{x}$

(1.5) $\lim_{x\to +\infty} \frac{\cos(3x)}{x} \xleftarrow{\text{squeeze}} \frac{[-1,1]}{x} \to 0$

(1.6) $\lim_{x\to -\infty} e^x \sin(x)$ $\quad e^{-N}\sin(N) = \frac{\sin(N) \leftarrow \text{bounded} \in [-1,1]}{e^N \leftarrow \text{large}} \longrightarrow \boxed{0}$

(1.7) $\lim_{x\to 0} e^x \sin(x) \overset{\text{D.S.}}{=\!=} e^0 \cdot \sin(0) = 1 \cdot 0 = 0$

(1.8) $\lim_{x\to 1} \left[\frac{1}{x-1} + \frac{4}{x^2-6x+5}\right]$

2. Use the definition of the derivative to compute $f'(x)$.

(2.1) $f(x) = \dfrac{1}{x}$

$$\lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{\frac{x}{x}\left(\frac{1}{x+h}\right) - \frac{1}{x}\left(\frac{x+h}{x+h}\right)}{h} = \lim_{h\to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{\frac{h}{1}}$$

$$= \lim_{h\to 0} \frac{\frac{-h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{-\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{-1}{x(x+h)} \overset{DS}{=} \frac{-1}{x^2}$$

(2.2) $f(x) = 2x^3$

$$\lim_{h\to 0} \frac{2(x+h)^3 - 2(x)^3}{h} = \lim_{h\to 0} \frac{2[(x+h)^3 - x^3]}{h}$$

$$2\lim_{h\to 0} \frac{(x+h)^3 - x^3}{h} = 2\lim_{h\to 0} \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 \cancel{- x^3}}{h}$$

1
1 1
1 2 1
1 3 3 1

degree sum = 3

$$= 2\lim_{h\to 0} \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$= 2(3x^2 + 3x(0) + 0^2) = 6x^2$$

3. Karen is talking on her phone again, and walking around aimlessly. Her phone tracks the distance from the car. These distances d are listed in the table, together with the time (in seconds) at which they were calculated.

t (seconds)	0	10	20	30	40	50
d (feet)	0	25	15	15	35	30

(3.1) Find Karen's average velocity for the time period beginning with $t = 10$ and lasting 10 seconds.

(3.2) Find Karen's average velocity for the time period beginning with $t = 20$ and lasting 10 seconds.

(3.3) Estimate their instantaneous velocity at $t = 20$ by finding the average rate of change from $t = 10$ to $t = 30$.

(3.4) Do you think this is a good estimate? Explain.

don't work

$\ln 2^2 + \overset{0}{\ln(1)} = \ln(4)$

DNE

$\ln(0), \ln(-1)$

$x = 0, -1, 2 \Rightarrow$

$2 \cdot \ln 2 + \ln(2-1) = \ln(2 \cdot 2)$

4. Find all solutions to

$$2\ln(x) + \ln(x-1) = \ln(2x)$$

Idea: ① get

$\ln(A) = \ln(B)$

② $e^{\ln(A)} = e^{\ln(B)}$

③ $A = B$ solve

① $\underbrace{\ln x^2 + \ln(x-1)} = \ln(2x)$

$\ln(x^2(x-1)) = \ln(2x)$

② $e^{\ln(x^2(x-1))} = e^{\ln(2x)}$

③ $x^2(x-1) = 2x$ poly.

④ $x^3 - x^2 - 2x = 0$

$x(x^2 - x - 2) = 0 \Rightarrow \overset{0}{x}\,\overset{0}{(x-2)}\,\overset{0}{(x+1)} = 0$

5. Given

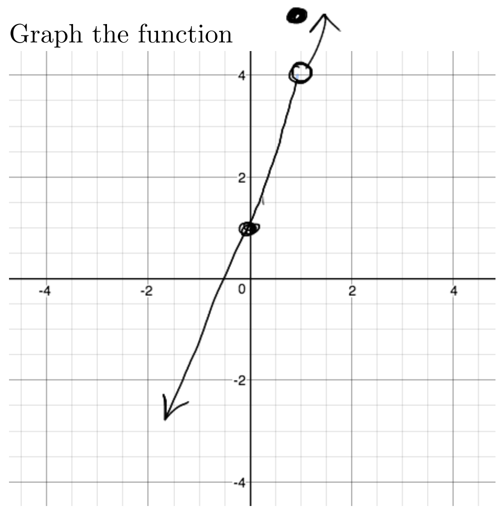
$$f(x) = \begin{cases} (x+1)^2 & x > 1 \\ 3x+1 & x < 1 \\ 5 & x = 1 \end{cases}$$

@ $x=1$ $(1+1)^2 = 4$

@ $x=1$ $3\cdot 1+1 = 4$

5

(5.1) Graph the function



(5.2) Finish the definition below:
A function $f(x)$ is continuous at $x = a$ if

$$\lim_{x \to a} f(x) = f(a)$$

(5.3) Use the definition of continuity to show that $f(x)$ is not continuous at $x = 1$.

$\boxed{\lim_{x \to 1} f(x) = 4}$ b/c Left limit: $\lim_{x \to 1^-} f(x) = \lim_{x \to 1} 3x+1 = 4$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} (x+1)^2 = 4$

but... $\boxed{f(1) = 5}$

(5.4) Is there a way to define $f(x)$ at $x = 1$ so that $f(x)$ is continuous at $x = 1$? Why or why not?

yes because limit exists if we redefine $f(x)$ @ $x=1$ to be 4 the result is continuous.

6. Simplify

$$\left(\frac{a^3b^3c^{-1}}{b^2c^{-2}a^4}\right)^{-5}$$

$$\left(\frac{bc}{a}\right)^{-5} = \left(\frac{a}{bc}\right)^{5} = \frac{a^5}{b^5c^5}$$

7. Simplify

$$\frac{\left(1+\frac{1}{x}\right)x}{\left(1-\frac{1}{x}\right)x} = \frac{x+\frac{x}{x}}{x-\frac{x}{x}} = \frac{x+1}{x-1}$$

common denominator

$$\frac{\frac{x}{x}\cdot 1 + \frac{1}{x}}{\frac{x}{x}\cdot 1 - \frac{1}{x}} = \frac{\frac{x}{x}+\frac{1}{x}}{\frac{x}{x}-\frac{1}{x}} = \frac{\frac{x+1}{x}}{\frac{x-1}{x}} = \frac{x+1}{\cancel{x}}\cdot\frac{\cancel{x}}{x-1} = \frac{x+1}{x-1}$$

8. (Give a short written response) What is the derivative of a function and what can you do with it?

Increasing & Decreasing:

$f(x) = x^3 - 8x$

$f'(x) = 3x^2 - 8 \quad = 0$

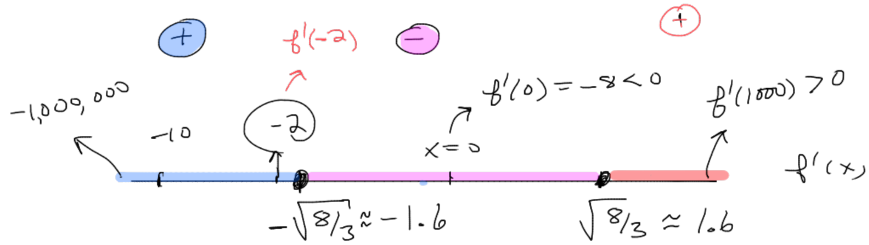
$3x^2 = 8$

$x^2 = \frac{8}{3}$

$x = \pm\sqrt{\frac{8}{3}} \implies$ critical points

$\implies$ f is increasing on $\oplus$ regions: $(-\infty, -\sqrt{8/3}) \cup (\frac{\sqrt{8}}{3}, \infty)$

decreasing on $(-\sqrt{8/3}, \sqrt{8/3})$



$f(x) = x^8 - 8x^7$

both dec. & concave up

$\downarrow$ $\downarrow$

$f'(x) < 0$ $f''(x) > 0$

$f'(x) = 8x^7 - 56x^6$

$= 8(x^7 - 7x^6) = 0$

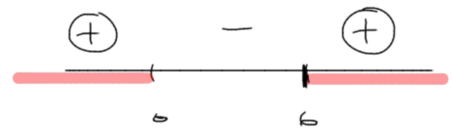
$8x^6(x-7) = 0$

C.P.'s $x = 0$, $x = 7$ $\rightarrow$

$f''(x) = 56x^6 - 6 \cdot 56x^5 = 0$

$56x^5(x-6) = 0$

$x = 0$, $x = 6$



$(6, 7)$