Monday - Week 5

1. e^x derivative
2. product rule

8. $f(x) = \frac{3}{2\sqrt{x}} + \frac{1}{\sqrt[3]{x}} - \frac{4}{\sqrt[4]{x}} = \frac{3}{2} \cdot x^{-\frac{1}{2}} + x^{-\frac{1}{3}} - 4x^{-\frac{1}{4}} - \frac{4}{4}$

$f'(x) = -\frac{1}{2} \cdot \frac{3}{2} x^{-\frac{3}{2}} - \frac{1}{3} x^{-\frac{4}{3}} + \frac{1}{4} \cdot 4x^{-\frac{5}{4}} = \boxed{-\frac{3}{4} x^{-3/2} - \frac{1}{3} x^{-4/3} + x^{-5/4}}$

power rule

10. $f(x) = \underbrace{3x\sqrt[3]{x}}_{\text{product isn't } x^m} - 7x + \frac{4}{\sqrt{x}} - 5e = 3x^{\frac{4}{3}} - 7x + 4x^{-\frac{1}{2}} - 5e$

$1 \quad 1/3$

$f'(x) = 4x^{\frac{1}{3}} - 7 - 2x^{-\frac{3}{2}}$

e^x _____

what is e?

$$\boxed{\lim_{x \to 0} \frac{e^x - 1}{x} = 1}$$

this 'really' is the definition of e.

$\lim_{x \to 0} \frac{a^x - 1}{x} = 1$ only when $a = e$.

what's the derivative of $f(x) = e^x$?

$$f'(x) = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h} = \lim_{h \to 0} \frac{e^x(e^h - 1)}{h} = e^x \cdot \lim_{h \to 0} \frac{e^h - 1}{h} = e^x \cdot 1$$

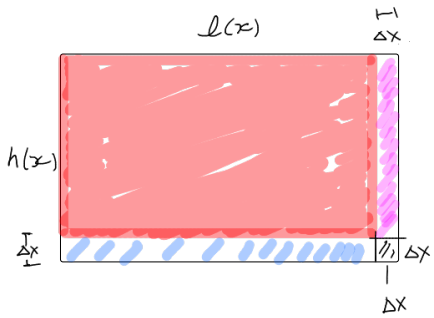
$$= e^x$$

def'n applied.

$$\boxed{\frac{d}{dx}(e^x) = e^x}$$

Products

think of two functions that depend on an indep. variable, eg time, & their product is something meaningful



views × watch-time = money

$\frac{d}{dt}(\text{money})$ = rate of increase/decrease of money

small change in $\ell(x)$

think: Δx is small $\Rightarrow$ $\Delta x^2 \approx 0$

New Area: 2 rectangles + small square = $\Delta x \cdot \ell(x) + \Delta x\, h(x) + \underbrace{\Delta x^2}_{\approx 0}$

small change in $h(x)$

$$\boxed{\text{Formula: } \frac{d}{dx}\big(f(x)\cdot g(x)\big) = f'(x)\cdot g(x) + f(x)\cdot g'(x)}$$

Ex $f(x) = e^x \cdot \cos x$ | $f'(x) = e^x \cdot \cos x + e^x(-\sin x) = e^x \cos x - e^x \sin x$

$= e^x(\cos x - \sin x)$

Ex $g(x) = (x^2+1)(x^3+x)$

$$g'(x) = \overbrace{(2x)(x^3+x)}^{2x^4+2x^2} + \overbrace{(x^2+1)(3x^2+1)}^{3x^4+x^2+3x^2+1} = 5x^4+6x^2+1$$

note: you could have done:

$$g(x) \overset{\text{algebra}}{=\!=\!=} x^5 + x^3 + x^3 + x$$

$$g'(x) = 5x^4 + 3x^2 + 3x^2 + 1 = 5x^4 + 6x^2 + 1$$