The wk 5

Power Rule

$(x+h)^2 = x^2 + 2xh + h^2$

$(x+h)^3 = x^3 + 3x^2h +$

$$\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}$$

$\binom{5}{1}$ $\binom{5}{2}$

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$

$\binom{n}{1} = n$

Recall, on Exam 1 # 2

$f(x) = 2x^3$, $f'(x) = 6x^2$
work

Let $f(x) = x^n$, $n \in \mathbb{Z}$ integer $\xrightarrow{\text{recall}}$ $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

$$f'(x) = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h\to 0} \frac{x^n + n\cdot x^{n-1}\cdot h + \binom{n}{2}x^{n-2}h^2 + \binom{n}{3}x^{n-3}h^3 + \dots + \binom{n}{n-1}xh^{n-1} + h^n - x^n}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}\left[nx^{n-1} + \binom{n}{2}x^{n-2}h + \binom{n}{3}x^{n-3}h^2 + \dots + \binom{n}{n-1}xh^{n-2} + h^{n-1}\right]}{\cancel{h}}$$

$$= \lim_{h\to 0} nx^{n-1} + \underbrace{\overset{\to 0}{\binom{n}{2}x^{n-2}h}}_{h\to 0} + \underbrace{\binom{n}{3}x^{n-3}h^2}_{\text{"} 0} + \dots + \binom{n}{n-1}xh^{n-2} + h^{n-1} = nx^{n-1}$$

Power Rule $\boxed{\frac{d}{dx}\left(x^n\right) = nx^{n-1} \quad \text{for any } n \in \mathbb{R}}$

derivative with respect to x

Ex 1

$f(x) = x^5$

$f'(x) = 5x^4$

$f(x) = \sqrt{x} = x^{1/2}$

$f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f(x) = x^{\pi}$

$f'(x) = \pi x^{\pi - 1}$

Properties of $\frac{d}{dx}$

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}\left(f(x)\right) \pm \frac{d}{dx}\left(g(x)\right)$$

(reason?

$$= \lim_{h \to 0} \left(\frac{f(x+h) + g(x+h) - (f(x) + g(x))}{h} \right) = \lim \frac{f(x+h) + g(x+h) - f(x) - g(x)}{h}$$

swap

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \boxed{\frac{g(x+h) - g(x)}{h}}$$

$$= f'(x) + g'(x)$$

Not true for $*$ $\frac{1}{2}$ $\div$

with addition and subtraction, you can take derivatives piece by piece

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$$\frac{d}{dx}\left(k(f(x))\right) = k \cdot \frac{d}{dx}\left(f(x)\right)$$

with derivatives, multiplicative constants come along for the ride

You can now differentiate any polynomial

$$f(x) = 3x^4 + 15x + 7 \cdot x^0$$

$$f'(x) = 3 \cdot 4x^3 + 15 + \underbrace{7 \cdot \underbrace{0 \cdot x^{-1}}_{=0}}_{=0} = 12x^3 + 15$$

Radicals & powers

$$f(x) = \frac{1}{x} + \sqrt[3]{x^2} \xrightarrow{\text{pre-process}} x^{-1} + x^{2/3}$$

$$f'(x) = -1 \cdot x^{-2} + \frac{2}{3}x^{2/3 - 1 \cdot \frac{3}{3}} = -x^{-2} + \frac{2}{3}x^{-1/3}$$

$$\boxed{\frac{d}{dx}\left(e^x\right) = e^x}$$

Key Fact

$$\boxed{\lim_{x \to 0} \frac{e^x - 1}{x} = 1} \longrightarrow \frac{d}{dx}\left(e^x\right) = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$e^{x+h} = e^x \cdot e^h$$

$$= \lim_{h \to 0} \frac{e^x\left(e^h - 1\right)}{h}$$

$$= e^x \cdot \underbrace{\lim_{h \to 0} \frac{e^h - 1}{h}} = e^x \cdot 1 = e^x$$