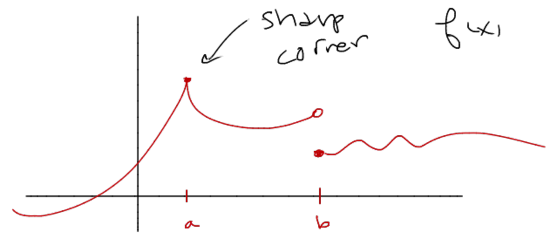
MA 161 - Week 5

▼1. 3.2
    a. power rule
    b. e^x rule
    c. sum rule
    d. constant rule
    e. parabola & boards
    f. power rule
    g. polynomials
▼2. 3.3
    a. quotient rule
    b. product rule
▼3. 3.4
    a. velocity
    b. graph interpretations
    c. optimization
▼4. 3.5
    a. higher derivatives
▼5. 3.6
    a. trig
    b. sin / cos / tan

The Derivative

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



$\Rightarrow$ Derivative, $f'(x)$ DNE @ $x = a$,
$x = b$

Power Rule

Idea: want a formula to compute derivatives of say,

$$f(x) = x^3$$

$$(f(x) = 2x^3$$

$$f(x) = 2x^3 + x^2 + \frac{1}{x}$$

From Exam 1

#2/ $f(x) = 2x^3$, $f'(x) = 6x^2$

In general

$f(x) = x^n \quad (n \in \mathbb{Z})$ ← integer 1, 2, 3, 4, ..

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{x^n + nx^{n-1}h + \binom{n}{2}x^{n-2}h^2 + \binom{n}{3}x^{n-3}h^3 + \dots + \binom{n}{n-1}xh^{n-1} + \binom{n}{n}h^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}\left[nx^{n-1} + \binom{n}{2}x^{n-2}h + \binom{n}{3}x^{n-3}h^2 + \dots + \binom{n}{n}h^{n-1}\right]}{\cancel{h}}$$

$$= \lim_{h \to 0} nx^{n-1} + \underbrace{\binom{n}{2}x^{n-2}h}_{=0} + \underbrace{\binom{n}{3}x^{n-3}h^2}_{=0} + \dots + \underbrace{\binom{n}{n}h^{n-1}}_{=0} = nx^{n-1}$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$\binom{5}{1}$ $\binom{5}{2}$ $\binom{5}{3}$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

derivative with respect to x

$$\boxed{\frac{d}{dx}\left(x^n\right) = nx^{n-1}}$$

... true for all $n \in \mathbb{R}$

Ex. $f(x) = x^5$

$f'(x) = 5x^4$

$f(x) = \sqrt{x}$
$= x^{1/2}$

$f'(x) = 1/2 \cdot x^{1/2 - 1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f(x) = x^{\pi}$

$f'(x) = \pi x^{\pi - 1}$

Next $\longrightarrow$ e^x

Key Fact: $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

So: If $f(x) = e^x$

$$f'(x) = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{\overbrace{e^x \cdot e^h - e^x}^{e^x(e^h - 1)}}{h}$$

$$= e^x \underbrace{\lim_{h \to 0} \frac{e^h - 1}{h}}_{=1} = e^x \cdot 1 = e^x$$

$$\boxed{\frac{d}{dx}\left(e^x\right) = e^x}$$

— Key Properties of d/dx

Since $\lim_{h\to 0} f(x) \pm g(x) = \lim_{h\to 0} f(x) \pm \lim_{h\to 0} g(x)$

and $\lim_{h\to 0} k \cdot f(x) = k \cdot \lim_{h\to 0} f(x)$ then

so, when things are added, you can take derivatives piece by piece

$$\frac{d}{dx}\left(f(x) \pm g(x)\right) = \frac{d}{dx}\left(f(x)\right) \pm \frac{d}{dx}\left(g(x)\right)$$

(note: not true for $*$ and $\div$)

for derivatives, multiplicative constants come along for the ride

$$\frac{d}{dx}\left(k \cdot f(x)\right) = k \cdot \frac{d}{dx}\left(f(x)\right)$$

Ex

$f(x) = 3x + \frac{1}{x} = 3x^1 + x^{-1}$

$f'(x) = 3 \cdot 1 \cdot x^{\overbrace{1-1}^{=0}} + -1x^{-2}$

$= 3 \cdot 1 \cdot 1 - x^{-2} = 3 - x^{-2}$

$f(x) = \frac{x^2 + 5x}{x} = x + 5$

(turn into a poly)

$f'(x) = 1$

$f(x) = 5\sqrt{x} + 100x^3$

$\quad = 5x^{\frac{1}{2}}$

$f'(x) = \frac{5}{2}x^{-\frac{1}{2}} + 300x^2$

$f(x) = 5 \cdot 1$

$= 5x^0$

(what is n?)

$f'(x) = 5 \cdot \underbrace{0}_{=0} \cdot x^{-1} = 0$