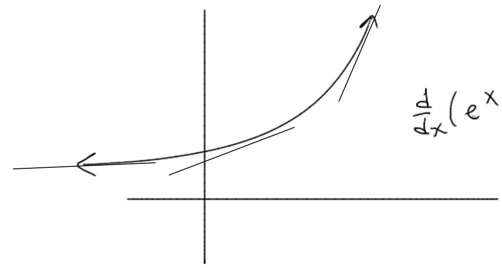
Wk 5 - Thur.

warm-up: $f(x) = \dfrac{e^x + \cos(x)}{3x^2 + \frac{1}{x}}$



$\frac{d}{dx}(e^x) = e^x$

$$f'(x) = \frac{(3x^2+\frac{1}{x})(e^x - \sin(x)) - (e^x + \cos(x))(6x - \frac{1}{x^2})}{(3x^2+\frac{1}{x})^2}$$

WW #19 /

Find an equation for the line tangent to the graph of f at $(2, 22)$, where f is given by $f(x) = 4x^3 - 4x^2 + 6$.

$y =$ 32x - 42

power rule

Line needs 2 ingredients:

point + slope

(2,22) (tangent) $\Rightarrow f'(x)$ @ $x = 2$

$x_1 = 2$, $y_1 = 22$ i.e., $m = f'(2)$

$f'(x) = 12x^2 - 8x$

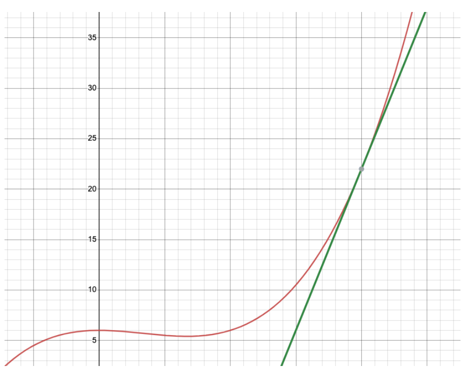
$f'(2) = 12 \cdot 4 - 8 \cdot 2 = 48 - 16 = 32 = m$

Formula for a line

$y - y_1 = m(x - x_1)$

$y - 22 = 32(x - 2)$

$y = 32x - 42$



18/

Find the derivative of $V = \frac{3}{2}\pi r^6 b$. Assume that b is a constant.

$\frac{dV}{dr} =$ $9\pi b r^5$

the derivative of V, assuming r is the variable (the derivative of V with respect to r

$\frac{dV}{dr} = 6 \cdot \frac{3}{2}\pi b r^5 = 9\pi b r^5$

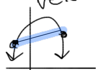
$V = \frac{3}{2}\pi r^6 b = \left(\frac{3}{2}\pi b\right) r^6$

similar to

$V = k \cdot r^6$

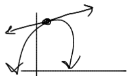
$\frac{dV}{dr} = 6 k r^5$

average velocity



secant line

instantaneous velocity



tangent line

parabola: 

22/

At a time t seconds after it is thrown up in the air, a tomato is at a height (in meters) of $f(t) = -4.9t^2 + 55t + 1$ m.

A. What is the average velocity of the tomato during the first 4 seconds? (Include units.) $(f(4) - f(0))/(4-0)$

B. Find (exactly) the instantaneous velocity of the tomato at $t = 4$. (Include units.) $f'(4)$

C. What is the acceleration at $t = 4$? (Include units.) $f''(4)$

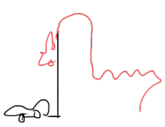
D. How high does the tomato go? (Include units.)

E. How long is the tomato in the air? (Include units.)

For part (c) recall what the derivative is

The derivative of a function $f(x)$ is the instantaneous rate of change of $f(x)$ with respect to x.

i.e., If $f(t) =$ position @ time t



$f'(t) =$ rate of change of position w.r.t. time. = velocity @ time t

$f''(t) =$ rate of change of velocity wrt time. = accelleration @ time t

$f'''(t) =$ rate of change of accelleration wrt time = jerk

(d) How high does tomato go?

Solve: $f'(x) = 0$ } this tells you when it was highest

then: plug in that time into original (height) function.

$f(t) = -4.9t^2 + 55t + 1$

└ initial height

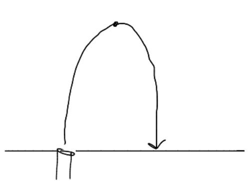
$f'(t) = -9.8t + 55$ (power rule)

set $f'(t) = 0 = -9.8t + 55 \Rightarrow t = \frac{55}{9.8} \approx$ 5.7 seconds

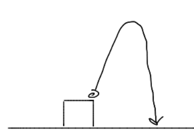
Finally max height $= f(55/9.8) =$

(e) How long is it in the air.

$at^2 + bt + c = 0$



ground



Solve $f(t) = 0$

$-4.9t^2 + 55t + 1 = 0$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$