thur. wk 5

$x^{-1} \rightsquigarrow -1x^{-2}$

warm-up!

$f(x) = (\underbrace{5x^3 + \tfrac{1}{x}}_{1st})\cdot(e^x + \sqrt{x})$

$f'(x) = (15x^2 - x^{-2})(e^x + \sqrt{x}) + (5x^3 + \tfrac{1}{x})(e^x + \tfrac{1}{2}x^{-1/2})$

$g(x) = \dfrac{(e^x + x)}{(\sqrt{x} - 1)}$

$g'(x) = \dfrac{(\sqrt{x} - 1)(e^x + 1) - (e^x + x)\tfrac{1}{2}x^{-1/2}}{(\sqrt{x} - 1)^2}$

Applications of the Derivative

Pure Math

$f(x) = 3x+7$, $f'(x) = 3$

$f(x) = -16x^2 + 50x + 100$

$f'(x) = -32x + 50$

$f''(x) = -32$

Application

$f(x)=3x+7$ give height (feet) at time x (sec)
units ↑ ft

$f'(x) = 3$ ft/sec ($\Rightarrow$ velocity = position/time)
|velocity| = speed

$f(x) = -16x^2 + 50x + 100$ gives the height (feet) of a ball thrown upward from 100' high building (on Earth), x seconds after it's been thrown.

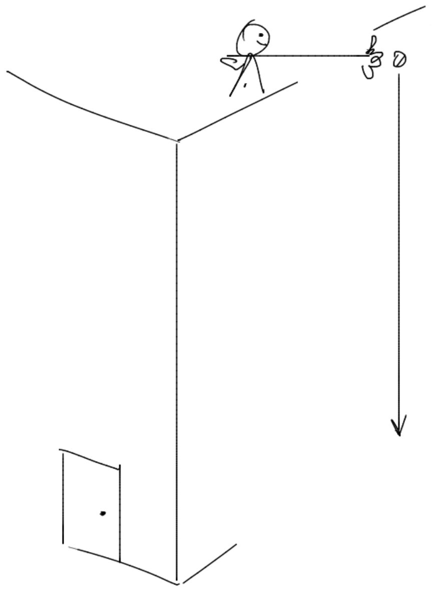
$\boxed{\text{Initial Position}}$: start height $\Leftrightarrow x = 0 \Rightarrow f(0) = -16 \cdot 0^2 + 50 \cdot 0 + 100$

$\boxed{f(0) = 100 \text{ ft}}$

$f'(x) = -32x + 50$ s

$\boxed{\text{Initial Velocity}}$: velocity upon release $(x=0) \Rightarrow \boxed{f'(0) = 50 \text{ ft/sec}}$

$f''(x) = -32 \text{ ft/sec}^2 = \frac{d}{dx}(\text{velocity})$ = accelleration



Suppose you throw a tomato down @ 25 ft/sec from a 100' building

(1) When does it hit ground?

(2) How fast is going upon impact?

(3) What if we throw it up instead?

Need: Function describing height:

$s(t) = -16t^2 - 25t + 100$ describes height above ground @ time t.

(1) set $s(t) = 0 = -16t^2 - 25t + 100$

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{25 \pm \sqrt{25^2 + 4 \cdot 1600}}{-32} = 1.83$ sec

(2) Impact velocity? $s(t) = 0 \leftrightarrow 1.83$ sec

$v(t) = s'(t) = -32t - 25$ $\Rightarrow$ Speed @ Impact ≈ 80–90 ft/sec

$v(1.83) \approx -32(2) - 25 = -64 - 25 = -89$