Wednesday - Wk 5

1. Warm-up (product rule) $\longrightarrow$ $\boxed{(f\cdot g)' = f'\cdot g + f\cdot g'}$
2. Quotient rule

Warm-up

$$f(x) = \left(x^2 + \frac{3}{x}\right)\cos(x) = \underbrace{\left(x^2 + 3x^{-1}\right)}_{\text{first}}\underbrace{\cos(x)}_{\text{2nd}}$$

$$f'(x) = \left(2x - 3x^{-2}\right)\cos(x) + \left(x^2 + 3x^{-1}\right)\left(-\sin(x)\right)$$

$$= \boxed{\left(2x - \frac{3}{x^2}\right)\cos(x) - \left(x^2 + \frac{3}{x}\right)\left(\sin(x)\right)}$$

Next, quotients!

Ex $f(x) = \dfrac{1}{x+1}$

power rule doesn't apply: $\dfrac{1}{x+1} = (x+1)^{-1}$ not exactly x^m

Quotient Rule

what is $f'(x)$ if $f(x) = \dfrac{F(x)}{G(x)}$?

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\dfrac{F(x+h)}{G(x+h)} - \dfrac{F(x)}{G(x)}}{h} = \lim_{h\to 0} \frac{1}{h}\left[\frac{F(x+h)}{G(x+h)} - \frac{F(x)}{G(x)}\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{F(x+h)}{G(x+h)} \cdot \frac{G(x)}{G(x)} - \frac{F(x)}{G(x)} \cdot \frac{G(x+h)}{G(x+h)}\right] = \lim_{h\to 0} \frac{1}{h} \frac{F(x+h)G(x) - F(x)G(x+h)}{G(x+h)G(x)}$$

$$= \lim_{h\to 0} \frac{1}{h} \frac{F(x+h)G(x) - F(x)\cdot G(x) + F(x)G(x) - F(x)G(x+h)}{G(x+h)G(x)}$$

$$a\,\frac{b-c}{d} = \frac{ab-ac}{d}$$

$$= \lim_{h\to 0} \frac{1}{h} \frac{G(x)\big(F(x+h) - F(x)\big) - F(x)\big(G(x+h) - G(x)\big)}{G(x+h)G(x)}$$

$$= \lim_{h\to 0} \frac{G(x)\dfrac{(F(x+h) - F(x))}{h} - F(x)\dfrac{(G(x+h) - G(x))}{h}}{G(x+h)G(x)}$$

$$= \frac{G(x)\overbrace{\lim_{h\to 0}\frac{(F(x+h) - F(x))}{h}}^{F'(x)} - F(x)\overbrace{\lim_{h\to 0}\frac{(G(x+h) - G(x))}{h}}^{G'(x)}}{\lim_{h\to 0} G(x+h)G(x) \xrightarrow[\text{direct sub}]{}} = \frac{G(x)\cdot F'(x) - F(x)\cdot G'(x)}{(G(x))^2}$$

$$\left(\frac{F(x)}{G(x)}\right)' =$$

quotient rule $\downarrow$

Remember

$$\left(\frac{Hi}{Lo}\right)' = \frac{Lo \cdot dHi - Hi\, d\,Lo}{Lo\,Lo}$$

Ex. $f(x) = \dfrac{x+3}{\sqrt{x}}$

$\sqrt{x} = x^{\frac{1}{2}} \xrightarrow{d/dx} \frac{1}{2}x^{-1/2}$

$$f'(x) = \frac{\sqrt{x}\cdot 1 - (x+3)\cdot \frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x})^2} = \frac{\sqrt{x} - \frac{x+3}{2\sqrt{x}}}{x}$$