Wed wk 5

warm-up

① $f(x) = 5x^3 + \frac{1}{x}$ ($= x^{-1}$), $f'(x) = 5\cdot 3x^2 + -1x^{\overset{-2}{-1-1}} = 15x^2 - x^{-2} = 15x^2 - \frac{1}{x^2}$

② $g(x) = e^x - \sqrt{x}$ ($= -x^{\frac{1}{2}}$), $g'(x) = e^x - \frac{1}{2}x^{-\frac{1}{2}}$

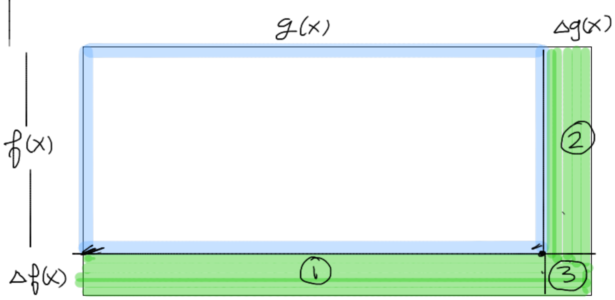
Today (3.3): Product & Quotient Rules

Products

Milkshakes, PB+J's

D=M·C,

How does $f(x)\cdot g(x)$ change? $\equiv$ $\frac{d}{dx}\left(f(x)\cdot g(x)\right) =$



as x changes, so does f(x), g(x)

$\Rightarrow$ $f(x)\cdot g(x)$ changes too

New area

Green Bit $= \Delta\left(f(x)\cdot g(x)\right) = ① + ② + ③ = g(x)\cdot\Delta f(x) + f(x)\cdot\Delta g(x) + \Delta f(x)\cdot\Delta g(x)$ $\to 0$ as $\Delta x \to 0$

To get the derivative, imagine $\Delta x \to 0$ (it's the "h" in def'n)

as $\Delta x \to 0$

$$\boxed{\frac{d}{dx}\left(f(x)\cdot g(x)\right) = \underbrace{g(x)\cdot\frac{d}{dx}\left(f(x)\right)}_{①} + \underbrace{f(x)\frac{d}{dx}\left(g(x)\right)}_{②}}$$ product rule

$\frac{d}{dx}\left(\text{first} \times \text{second}\right) =$ derivative of first times the second + derivative of the second times first

Ex

$f(x) = (3x+1)(x^2+x)$

$f'(x) = 3\cdot(x^2+x) + (3x+1)(2x+1)$

$= 3x^2+3x + 6x^2+3x+2x+1$

$= \underline{9x^2+8x+1}$

could've rewrite original

FOIL $= 3x^3+3x^2+x^2+x$

$\downarrow d/dx$

$9x^2+6x+2x+1$

$\underline{9x^2+8x+1}$

Ex

$g(x) = e^x\left(5x+\frac{1}{x}\right)$

$g'(x) = e^x\left(5x+\frac{1}{x}\right) + e^x\left(5-\frac{1}{x^2}\right)$

deriv. 1st

deriv. of 1st

copy 2nd

deriv 2nd

you can even do triple products

$$\frac{d}{dx}\Big(\underbrace{f(x)\cdot g(x)}_{\text{Imagine as 1 function (First)}}\cdot \overbrace{k(x)}^{2nd}\Big) = \frac{d}{dx}(\text{First}) \times 2nd + \frac{d}{dx}(2nd) \times \text{First}$$

$$= \underbrace{\frac{d}{dx}\big(f(x)\cdot g(x)\big)}\cdot k(x) + k'(x)\cdot\big(f(x)\cdot g(x)\big)$$

$$= \big[f'(x)\cdot g(x) + g'(x) f(x)\big] k(x) + k'(x)\cdot f(x)\cdot g(x)$$

distribute

$$= f'(x)\, g(x)\, k(x) + f(x)\, g'(x)\, k(x) + f(x)\, g(x)\, k'(x)$$

Quotient Rule

$$\boxed{\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)} = \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)}\cdot\frac{g(x)}{g(x)} - \frac{f(x)}{g(x)}\cdot\frac{g(x+h)}{g(x+h)}}{h}$$

common denom

$$= \lim_{h\to 0} \frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)g(x)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{1}{g(x+h)g(x)} \cdot \frac{f(x+h)g(x) - f(x)g(x+h)}{h}$$

$$= \lim_{h\to 0} \frac{1}{g(x+h)g(x)} \left[\frac{f(x+h)g(x) \overbrace{- g(x)f(x) + f(x)\cdot g(x)}^{\text{add } 0} - f(x)g(x+h)}{h}\right]$$

= factor by grouping get

$$\lim_{h\to 0} \frac{1}{g(x+h)g(x)} \left[g(x)\lim_{h\to 0}\frac{[f(x+h)-f(x)]}{h} - f(x)\lim_{h\to 0}\frac{[g(x+h)-g(x)]}{h}\right]$$

$$= \frac{1}{g^2}\cdot\left[g\cdot f' - f\cdot g'\right]$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \boxed{\frac{g(x)f'(x) - f(x)\cdot g'(x)}{(g(x))^2}}$$

$$\frac{d}{dx}\left(\frac{hi}{lo}\right) =$$ ho dee hi minus hi dee lo over lo lo