Wed wk 5

warm-up

$f(x) = 5x + \frac{1}{x}$ $\quad \frac{1}{x} = x^{-1}$

$f'(x) = 5 \cdot \underbrace{1x^{0}}_{=1} + (-1)x^{-1-1} = 5 - x^{-2} = 5 - \frac{1}{x^2}$

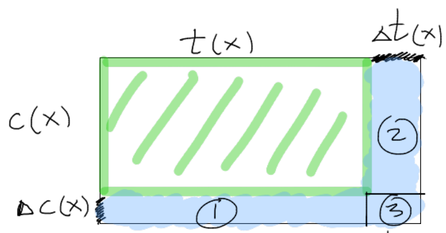
$g(x) = e^x + 1 + 7x^6$ $\qquad g'(x) = e^x + 0 + 7 \cdot 6x^5 = e^x + 42x^5$

Today: Product & Quotient Rule

Idea behind product rule:

$F(x) = f(x) \cdot g(x)$

Famous
- cookies
- quesadillas
- ice cream
- human life



Area ② $\Delta t(x) \cdot c(x)$

Area $= c(x) \cdot t(x)$

Suppose x changes, thus $c(x)$ change & $t(x)$ change

so $c(x) \cdot t(x)$ changes too,

Area ① $\Delta c(x) \cdot t(x)$

Area ③ $\Delta c(x) \cdot \Delta t(x)$

New area $= \Delta(c(x) \cdot t(x))$

as $\Delta x \to 0$ section ③ $\to 0$ b/c small # $\cdot$ small # $\approx$ very small

as $\Delta x \to 0$ ① $\to c'(x) \cdot t(x)$

② $\to t'(x) \cdot c(x)$

$$\Delta(c(x) \cdot t(x)) = \Delta c(x) \cdot t(x) + \Delta t(x) \cdot c(x) + \Delta c(x) \Delta t(x)$$

as $\Delta x \to 0$

$$\boxed{\frac{d}{dx}\left(c(x) \cdot t(x)\right) = c'(x) \cdot t(x) + t'(x) \cdot c(x)} + 0$$

Exercises

① $(2x+1)(3x^2+x) \xrightarrow[d/dx]{\text{product rule}} 2\cdot(3x^2+x) + (2x+1)(6x+1)$

$6x^2+2x + 12x^2+2x+6x+1$

$= 18x^2+10x+1$

FOIL $\|$

$6x^3+2x^2+3x^2+x$

$\big\downarrow d/dx$

$18x^2+4x+6x+1$

$18x^2+10x+1$

same

② $f(x) = (e^x+x^2+3x)(x^{-1}-5x^4)$

$f'(x) = (e^x+2x+3)(x^{-1}-5x^4) + (e^x+x^2+3x)(-x^{-2}-20x^3)$

Quotient Rule

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)g(x)}{g(x+h)g(x)} + \frac{f(x)g(x)}{g(x+h)g(x)} - \frac{f(x)}{g(x)}}{h}$$

we added 0

common denom & split

$$= \lim_{h\to 0} \frac{\frac{f(x+h)}{g(x+h)}\cdot\frac{g(x)}{g(x)} - \frac{f(x)g(x)}{g(x+h)g(x)}}{h} + \frac{\frac{f(x)g(x)}{g(x+h)g(x)} - \frac{f(x)}{g(x)}\cdot\frac{g(x+h)}{g(x+h)}}{h}$$

factor by grouping

$$= \lim_{h\to 0} \underline{g(x)\left[\frac{f(x+h)-f(x)}{g(x+h)\cdot g(x)}\right]}\frac{1}{h} - \lim \underline{f(x)\left[\frac{g(x+h)-g(x)}{g(x+h)g(x)}\right]}\frac{1}{h}$$

$$= g(x)\cdot\lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]\cdot\frac{1}{g(x+h)g(x)} - f(x)\cdot\lim_{h\to 0}\left[\frac{g(x+h)-g(x)}{h}\right]\frac{1}{g(x+h)g(x)}$$

$$= \frac{g(x)\cdot f'(x)}{g(x)\cdot g(x)} - \frac{f(x)g'(x)}{g(x)\cdot g(x)} = \boxed{\frac{g(x)\cdot f'(x) - f(x)g'(x)}{(g(x))^2}}$$

$$\frac{d}{dx}\left(\frac{hi}{lo}\right) = lo\cdot d\,hi \text{ minus } hi\, d\, lo \text{ over } lo$$

$$f(x) = \frac{3x^2 + e^x + 5x}{2x+1}$$

$$f'(x) = \frac{(2x+1)(6x + e^x + 5) - (3x^2 + e^x + 5x)(2)}{(2x+1)^2}$$

<u>stop</u>!