Friday - Week 6

1. Exam study guide up
2. Derivative Chart - Draft # 1
3. Implicit Differentiation
    a. ln(x)
    b. inverse trig
    c. x^x

warm-up #9

$$y = \frac{\sqrt{x}-1}{\sqrt{x}+1} \quad \begin{matrix}\text{Hi}\\ \text{Lo}\end{matrix}$$

$$\sqrt{x}-1 = x^{\frac{1}{2}}-1 \quad \downarrow d/dx$$

$$y' = \frac{(\sqrt{x}+1)\frac{1}{2}(x)^{-\frac{1}{2}} - (\sqrt{x}-1)(\frac{1}{2}x^{-\frac{1}{2}})}{(\sqrt{x}+1)^2} = \frac{\frac{1}{2}(x^{-\frac{1}{2}})\overbrace{[(\sqrt{x}+1) - \underbrace{(\sqrt{x}-1)}_{-\sqrt{x}+1}]}^{2}}{(\sqrt{x}+1)^2} = \frac{2}{\sqrt{x}(\sqrt{x}+1)^2}$$

Derivative Chart - Draft # 1

$u = f(x)$

Functions	Derivatives
u^n $(n \in \mathbb{R})$	$n \cdot u^{n-1} \cdot \frac{du}{dx}$
$\sin(u)$	$\cos(u) \cdot \frac{du}{dx}$
$\cos(u)$	$-\sin(u) \frac{du}{dx}$
$\tan(u)$	$\sec^2(u) \frac{du}{dx}$
$\csc(u)$	$-\csc(u)\cot(u) \frac{du}{dx}$
$\sec(u)$	$\sec(u)\tan(u) \frac{du}{dx}$
$\cot(u)$	$-\csc^2(u) \frac{du}{dx}$
e^u	$e^u \cdot \frac{du}{dx}$

Functions	Derivatives
$\ln(u)$	$\frac{1}{u} \cdot \frac{du}{dx}$
$\sin^{-1}(u)$	$\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$

Implicit Differentiation

(chain rule) $\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$ → deriv. inside

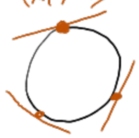
└ deriv. outside

Notation:

$\frac{dy}{dx} = y'$ — "y-prime"

└ deriv. of y wrt. x

Equation of Circle, radius $= 1$



To find $\frac{dy}{dx}$, realize: $\frac{dy}{dx} = \frac{d}{dx} y$ & $\frac{d}{dx}(x) = \frac{dx}{dx} = 1$

$$x^2 + y^2 = 1$$

$\downarrow$ apply $\frac{d}{dx}$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \underbrace{\frac{d}{dx}(1)}$$

$$2x + 2y \cdot y' = 0$$

w/ regards to chart

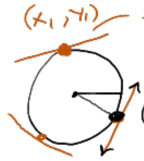
$$\frac{d}{dx}(x^2) = 2x \cdot \frac{dx}{dx} = 2x$$

$$\frac{d}{dx}(y^2) = 2y \cdot \frac{dy}{dx} = 2y \cdot y'$$

get a y' multiplied whenever the variable DOES NOT MATCH the what your taking the derivative with respect to

solve for y'

$$2y\,y' = -2x \implies \boxed{y' = \frac{-2x}{2y} = -\frac{x}{y}}$$



$(.15, -.1)$, slope of tan line $= \frac{-(.15)}{-.1} = .015$

$f(x) = \ln(x)$

1. set $y = \ln(x)$
2. raise both sides as powers of e. $e^y = e^{\ln(x)} = x$
3. use implicit diff to find y'.

$$\frac{d}{dx}(e^y) = \frac{d}{dx}(x)$$

look @ chart
$y \longleftrightarrow u$

$$e^y \frac{dy}{dx} = 1$$

replace w/ y' if you wish

4. Isolate $\frac{dy}{dx} = \frac{1}{e^y}$
5. get original var. back. (use 1)

$$\frac{dy}{dx} = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{x}}$$

So with chain rule

$$\frac{d}{dx}(\ln(u)) = \underbrace{\frac{d}{du}(\ln(u))}_{\frac{1}{u}} \cdot \frac{du}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$$

deriv. of inside

Ex

$y = \ln(x^2 + 5)$ $\quad u = x^2 + 5 \rightarrow \frac{1}{u} \cdot$ (deriv. of u)

$y' = \left(\frac{1}{x^2+5}\right)(2x) = \frac{2x}{x^2+5}$

Ex

$f(x) = \ln(\sin(x))$

$f'(x) = \frac{1}{\sin(x)} \cdot \cos(x) = \frac{\cos(x)}{\sin(x)} = \cot(x)$

Tip: It's just U' / U or derivative over original

Ex

$f(x) = \ln(e^{x^2}) = x^2$

power

$f'(x) = \frac{e^{x^2} \cdot 2x}{e^{x^2}} = 2x$

$e^u \rightarrow e^u \cdot \frac{du}{dx}$

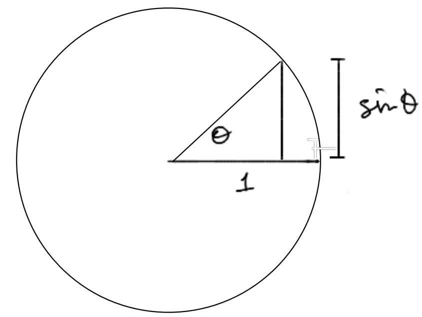
$e^{x^2} \rightarrow e^{x^2} \cdot 2x$

why is this derivative so simple ^^^^

- The derivative of inverse trig functions: $f(x) = \sin^{-1}(x)$

Recall, $y = \sin(\theta)$ = y-coord of point det. by angle θ

= height of point @ angle θ.



So

$\theta = \sin^{-1}(y)$

thus θ = angle that gives point of height y.

so arcsin(x) is just an angle

Follow same process as for $\ln(x)$:

① $y = \sin^{-1}(x)$

② $\sin(y) = x$

③ $\frac{d}{dx} \rightarrow \quad \frac{d}{dx}(\sin(y)) = \frac{d}{dx}(x)$

chart

$\cos(y) \cdot \frac{dy}{dx} = 1$

④ $\frac{dy}{dx} = \frac{1}{\cos(y)}$

⑤ get x back in answer

relate $\cos(y)$ w/ $\sin(y)$, use ②

$\cos^2(y) + \sin^2(y) = 1$

$\cos^2(y) = 1 - \sin^2(y)$

$\cos(y) = \pm\sqrt{1 - \sin^2(y)}$

(from the def'n of $\cos^{-1}(x)$ that here:

$\cos(y) = \sqrt{1 - \sin^2(y)}$

$= \sqrt{1 - x^2}$ step ②

⑥ $\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$

So w/ chain rule:

$$\boxed{\frac{d}{dx}\left(\sin^{-1}(u)\right) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}}$$