Mon, Wk 6

#6 WW

Find the equations of the tangent lines to the curve $y = \frac{x-4}{x+4}$ that are parallel to the line $x - 8y = 2$.

$y =$ ______

, $y =$ ______

$\frac{1}{8}x - \frac{1}{4} = y$

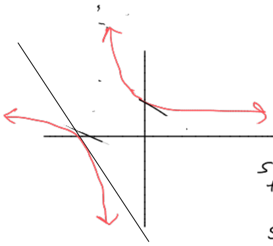
$x - 2 = 8y$

$x = 2 + 8y$

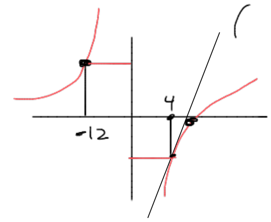
$\downarrow$

$y = \frac{1}{8}x - \frac{1}{4}$

$\boxed{\text{slope} = \frac{1}{8}}$



slope of tangent // slope $= \frac{1}{8}$



Get equation for slope of tan. line, set it equal to $\frac{1}{8}$, solve.

take derivative

$$\frac{d}{dx}(y) = \frac{(x+4)\cdot 1 - (x-4)\cdot 1}{(x+4)^2} = \frac{1}{8}$$

$-x+4$

$$= \frac{8}{(x+4)^2} = \frac{1}{8}$$

$64 = (x+4)^2$

$\pm\sqrt{64} = x+4$

$-4 \pm 8 = x$ x-coordinates, each has a coorespondins y-coord

$-12 = x$ or $x = 4$

$x = -12 \Rightarrow y = \frac{-12-4}{-12+4} = \frac{-16}{-8} = 2$

$x = 4 \Rightarrow y = 0$

$(-12, 2)$

$(4, 0)$

$\leftarrow$ combine to get two equations of lines w/ $m = \frac{1}{8}$

$\boxed{y - y_1 = m(x - x_1)}$

$m = \frac{1}{8}$

$(x_1, y_1) = (-12, 2)$

$y - 2 = \frac{1}{8}(x + 12)$

$m = \frac{1}{8}$

$(x_1, y_1) = (4, 0)$

$y - 0 = \frac{1}{8}(x - 4)$

Quotient Rule

$$\frac{\text{lo dee hi} - \text{hi dee lo}}{\text{lolo}}$$

$$f(x) = \frac{(t+2)(4t^3+1)}{(t^2-4)(t-1)}$$

$$f'(x) = \frac{(t^2-4)(t-1)\left[1\cdot(4t^3+1) + (t+2)(12t^2)\right] - (t+2)(4t^3+1)\left[2t(t-1) + (t^2-4)\cdot 1\right]}{\left((t^2-4)(t-1)\right)^2}$$

$$f(t) = \underbrace{(t^2+2t+3)}_{\text{first}}\underbrace{(4t^{-2}+5t^{-3})}_{\text{second}}$$

$$f'(t) = d(\text{first})\cdot\text{second} + \text{first}\cdot d(\text{second})$$

$$= [2t+2][4t^{-2}+5t^{-3}] + [t^2+2t+3][-8t^{-3}-15t^{-4}]$$

12/

Find $\frac{d}{ds}\left(\frac{1}{s+ke^s}\right)$.

k = constant

s = variable

$\frac{d}{dx}(x) = \frac{dx}{dx} = 1$

power

- power
- product
- quotant ☆
- $e^x \longmapsto e^x$

$$= \frac{(s+ke^s)\cdot 0 - 1\cdot(1+ke^s)}{(s+ke^s)^2}$$

$$= \frac{-(1+ke^s)}{(s+ke^s)}$$

Chain Rule

Suppose: $F(x) = f(g(x))$ composite function

chain rule tells us how to find $\frac{d}{dx}\left(F(x)\right)$

Idea:

$$\text{Feel}(\text{Eat}(\text{smaller food})) = F(e(s))$$

$$\frac{dF}{ds} = \frac{dF}{de} \cdot \frac{de}{ds}$$

So

$$\boxed{\underbrace{\frac{d}{dx}\left(F(G(x))\right)}_{\frac{dF}{dx}} = \underset{\frac{dF}{dG}}{\underset{\parallel}{F'(G(x))}} \cdot \underset{\frac{dG}{dx}}{\underset{\parallel}{G'(x)}}}$$

Ex

$$f(x) = (3x^2 + x)^2$$

$$f'(x) = \underbrace{2(3x^2+x)^1}_{\text{derivative of the outside evaluated at the inside}} \cdot \underbrace{(6x+1)}_{\text{derivative of the inside}}$$

$$= (12x+2)(3x^2+x)$$