Mon - wk 6

Last week: product rule, quotient rule, basic apps

Ex compute $f'(x), f''(x), f'''(x), f^{(4)}(x)$

$f(x) = (3x^2 + x)(5x+1)$

$f'(x) = (6x+1)(5x+1) + (3x^2+x)\cdot 5$

$= 30x^2 + 6x + 5x + 1 + 15x^2 + 5x$

$= 45x^2 + 16x + 1$

$f''(x) = \frac{d}{dx}(f'(x)) = \frac{d}{dx}\left((6x+1)(5x+1) + (3x^2+x)\cdot 5\right)$

$= 6\cdot(5x+1) + (6x+1)\cdot 5 + (6x+1)\cdot 5 + \underbrace{(3x^2+x)\cdot 0}_{=0}$

$= 30x + 6 + 30x + 5 + 30x + 5 = 90x + 16$

or

$\frac{d}{dx}(45x^2 + 16x + 1) = 90x + 16$

$f'''(x) = 90, \quad f^{(4)}(x) = 0$

Today: CHAIN RULE, Higher Derivatives

$f''(x) = \frac{d}{dx}(f'(x))$

$f'''(x) = \frac{d}{dx}(f''(x))$

why?

If $f(x)$ = position @ time

e.g., f = ft, x = time

then $f'(x) = \frac{d}{dx}(f(x))$ = velocity

$f''(x)$ = acceleration $= \frac{d}{dx}(f'(x))$

$f'''(x) = \frac{d}{dx}(f''(x))$ jerk

$A = \pi r^2$ area of circle

$$\frac{dA}{dr} = 2\pi r$$

Here r is constant

If r changes wrt time, $\frac{dA}{dt}$ will change

So $A = \pi (r(t))^2$

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

CHAIN RULE

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$$

Formal Chain Rule Def'n

$F(x) = f(g(x))$ 		$g(x)$ is "inside" $f(x)$

$F'(x) = \dfrac{df}{dx} = \underbrace{\dfrac{df}{dg}}_{\text{derivative of outside}} \cdot \dfrac{dg}{dx}$ derivative of inside

$x^3 \to 3x^2$

$(\text{blah})^3 \to 3(\text{blah})^2 \cdot \dfrac{d\,\text{blah}}{dx}$

Ex.

$F(x) = (3x+1)^3 = f(g(x))$ where $f(x) = x^3$

$g(x) = 3x+1$

$F'(x) = \dfrac{df}{dg} \cdot \dfrac{dg}{dx} = 3(3x+1)^2 \cdot 3$

derivative of x^3 evaluated @ g } $3x^2$ evaluated @ $x \to 3x+1$

$3(3x+1)^2$

Derivatives

So far, we know ________________________

- polynomials
- e^x
- products / quotients

Chain

Ex

$$f(x) = e^{(5x^2+3x+2)}$$

$$f'(x) = \underbrace{e^{5x^2+3x+2}}_{\text{deriv. of outside}} \cdot \underbrace{(10x+3)}_{\text{deriv. of inside}}$$

Ex

$$f(x) = 5(e^x+1)^2$$

$$f'(x) = 2\cdot 5(e^x+1)^1 \cdot e^x$$

$$= 10e^x(e^x+1)$$

Ex.

$$f(x) = \left(\frac{x+1}{\sqrt{x}-1}\right)^2$$

Ex

$$g(x) = \left((e^x-1)(x^2+1)+1\right)^5$$