Practice

$f(x)$ — $f'(x)$

① $(3x+4)^5$ — $5(3x+4)^4 \cdot 3$

② $e^{\sqrt{x}} \longrightarrow e^{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$

③ $\frac{1}{e^x} = e^{-x} \longrightarrow e^{-x} \cdot (-1) = -e^{-x}$

④ $\Big(\underbrace{\overbrace{(3x+4)}^{\text{1st}}\overbrace{(6\sqrt{x} + e^x)}^{\text{2nd}}}_{\text{inside is a product}}\Big)^5$

$6x^{1/2} \to \frac{1}{2}6x^{-1/2}$

$5\big((3x+4)(6\sqrt{x} + e^x)\big)^4 \cdot \left[3(6\sqrt{x} + e^x) + (3x+4)\left(\frac{3}{\sqrt{x}} + e^x\right)\right]$

Reminder / Tip

You've already (always) been using the chain rule:

$5x^4 \longrightarrow 5\cdot 4x^3$

chain rule

$5 \cdot 4x^3 \cdot 1$

Today

TRIG DERIVATIVES & their connection w/ chain rule

$$\frac{d}{dx}\left(\sin(x)\right) = \underline{\cos(x)}$$

$$\| $$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x)\cos(h)+\sin(h)\cos(x)-\sin(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x)[\cos(h)-1]+\sin(h)\cos(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin(x)[\cos(h)-1]}{h} + \lim_{h\to 0}\frac{\sin(h)\cos(x)}{h}$$

$$= \sin(x)\cdot\underbrace{\lim_{h\to 0}\frac{\cosh-1}{h}}_{=0} + \cos(x)\underbrace{\lim_{h\to 0}\frac{\sin(h)}{h}}_{=1} = \sin x\ 0 + \cos(x)\cdot 1 = \cos(x)$$

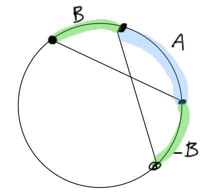
We'll need these tools:

$\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$

$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$

$$\lim_{h\to 0}\frac{\sin(h)}{h} = 1$$

$$\lim_{h\to 0}\frac{\cos(h)-1}{h} = 0$$



By the same reasoning: (use tool # 2)

$$\boxed{\frac{d}{dx}\left(\cos(x)\right) = \underline{-\sin(x)}}$$

$$\boxed{\frac{d}{dx}\left(\tan(x)\right) = \sec^2(x)}$$

why should we care about the derivative of tan(x)?

we care about how slopes change

Hint: Stuck dealing tan(x)??? …. re-write in terms of sin(x)/cos(x)

$$\frac{d}{dx}\left(\tan(x)\right) = \frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right) = \frac{\cos(x)\cdot\cos(x) - \sin(x)\cdot(-\sin(x))}{(\cos(x))^2} = \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)}$$

$$= \left(\frac{1}{\cos(x)}\right)^2 = \sec^2(x)$$

Chain Rule + Trig

— $f(x)$ —	— $f'(x)$ —
① $\cos(\sqrt{x})$	$-\sin(\sqrt{x})\cdot\frac{1}{2}x^{-1/2}$
② $\sin\left(\frac{x+1}{x^2-e^x}\right)$	$\cos\left(\frac{x+1}{x^2-e^x}\right)\cdot\frac{(x^2-e^x)-(x+1)(2x-e^x)}{(x^2-e^x)^2}$
③ $e^{\tan(x)}$	$e^{\tan(x)}\cdot\sec^2(x)$

$$\frac{d}{dx}(\sin(u)) = \cos(u)\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(\cos(u)) = -\sin(u)\cdot\frac{du}{dx}$$

$$\frac{d}{dx}(e^u) = e^u\cdot\frac{du}{dx}$$