Ex:

$$f(x) = \left(\frac{x+1}{\sqrt{x}-1}\right)^2$$

$$f'(x) = 2\left(\frac{x+1}{\sqrt{x}-1}\right) \cdot \frac{(\sqrt{x}-1)\cdot 1 - (x+1)\frac{1}{2}x^{-\frac{1}{2}}}{(\sqrt{x}-1)^2} = \frac{(2x+2)\left(\sqrt{x}-1 - \frac{x+1}{2\sqrt{x}}\right)}{(\sqrt{x}-1)^2}$$

$$h(x) = e^{3x+1} \qquad e^{x} \to e^{x}$$

$$h'(x) = e^{3x+1} \cdot 3$$

Ex

$$g(x) = \left((e^x-1)(x^2+1)+1\right)^5$$

$$g'(x) = 5\left(\underbrace{(e^x-1)}_{1^{st}}\underbrace{(x^2+1)}_{2nd}+1\right)^4 \cdot \left(e^x(x^2+1)+(e^x-1)(2x)\right)$$

Hint:
"you've been using chain rule all along"

$$g(x) = 5x^7$$

$$g'(x) = 5\cdot 7x^6 = 35x^6$$

$$g'(x) = 5\cdot 7x^6 \cdot 1 = 35x^6$$

(chain rule) ↑ deriv. of inside

BRIEF COMPARISON OF DERIVATIVE TYPES

(+) Your Income = Sum of Income from various sources

$$I(x) = J_1(x) + J_2(x) + h(x)$$

$$I'(x) = J_1'(x) + J_2'(x) + h'(x)$$

$J_1'(x)$: $+?$ $J_2'(x)$: $-?$ $h'(x)$: $=0$

$I'(x)$: increase/decrease amount of income

(*) Total Amount of stuff owned by humans $= N(x) \cdot C(x)$

Total Amount of stuff owned by humans $= A(x)$

$$A'(x) = N'(x) \cdot C(x) + N(x) \cdot C'(x)$$

$A'(x)$: increase/decrease

$N'(x)$: more/less

$C'(x)$: more stuff / less stuff

(∘) composition

$$\text{Climate Status} = S(x) = f(p(x), x) \approx f(p(x))$$

$p(x)$: Impact of People

$f(p(x), x)$: two variables

$$S'(x) = \underbrace{f'(p(x))}_{\text{how climate is changing (default) @ current level of human impact}} \cdot p'(x)$$

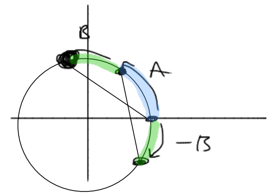
$S'(x)$: How climate changing

$f'(p(x))$: calculus

$\cdot$: multiplicative

$p'(x)$: change of people's impact

TRIG DERIVATIVES



TOOLS:

$$\sin(A+B) = \sin(A)\cos(B) + \sin(B)\cos(A)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\lim_{h\to 0} \frac{\sin(h)}{h} = 1 \quad , \quad \lim_{h\to 0} \frac{\cos(h)-1}{h} = 0$$

$$\boxed{\frac{d}{dx}\left(\sin(x)\right) = \cos(x)}$$

$$\parallel$$

$$\lim_{h\to 0} \frac{\sin(x+h) - \sin(x)}{h} \overset{\text{tool \#1}}{=} \lim_{h\to 0} \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x)[\cos(h)-1] + \sin(h)\cos(x)}{h} = \lim_{h\to 0} \frac{\sin(x)(\cos(h)-1)}{h} + \lim_{h\to 0} \frac{\sin(h)\cos(x)}{h}$$

$$= \sin(x) \cdot \underbrace{\lim_{h\to 0}\left(\frac{\cos(h)-1}{h}\right)}_{\text{tool \#4}} + \cos(x) \cdot \underbrace{\lim_{h\to 0} \frac{\sin(h)}{h}}_{\text{tool \#3}} = \sin x \cdot 0 + \cos(x)\cdot 1 = \cos(x)$$

By the <u>same reasoning</u> (use tool # 2)

$$\boxed{\frac{d}{dx}\left(\cos(x)\right) = -\sin(x)}$$

Hint: All "co-functions" have negative derivative.

$\cot(x)$, $\csc(x)$, ...

$$\boxed{\frac{d}{dx}\left(\tan(x)\right) = \sec^2(x)}$$

(why should we care?
→ describes

#1 rule w/ tangent : (stuck?)... re-write as $\frac{\sin(x)}{\cos(x)}$

$$\frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right) = \frac{\cos(x)\cdot\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

(quotient!

$$= \left(\frac{1}{\cos(x)}\right)^2$$

$$= \sec^2(x)$$

Exercises

$f(x) = \cos(\sqrt{x})$

$f'(x) =$

$f(x) = \cos(\sin(x))$

$f'(x) =$

$f(x) = \sin\left(\frac{1}{x}\right)$

$f'(x) =$

$f(x) = \tan\left(\frac{x+1}{1-\sqrt{x}}\right)$

$f'(x) =$

$f(x) = e^{\cos(\sqrt{x})}$

$f'(x) =$

$\frac{d}{dx}(\cos(u)) = -\sin(u)\cdot\frac{du}{dx}$

$\frac{d}{dx}(\sin(u)) = \cos(u)\cdot\frac{du}{dx}$