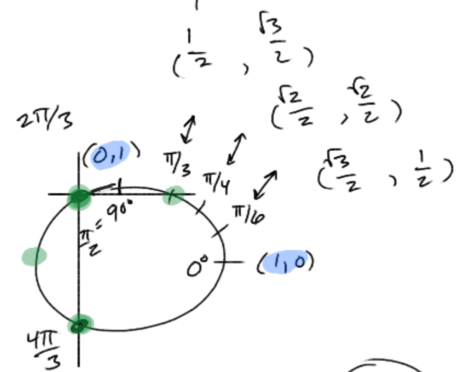
Thursday - Week 6

1. Any questions? → #14
    a. speak up or put in chat.
2. Derivative of tan(x)
    a. and other trig functions
3. Chain Rule motivation
4. Chain Rule examples

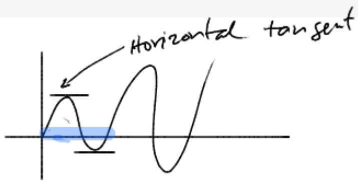
$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$



#14

For what values of x in $[0, 2\pi]$ does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent?
List the values of x below. Separate multiple values with commas.

$x =$ $\{2\pi/3, 4\pi/3\}$



① $f'(x) = 1 + 2\cos x$

② $f'(x) = 0 = 1 + 2\cos x \implies -\frac{1}{2} = \cos(x)$, $x = \operatorname{acos}(-\frac{1}{2}) = \frac{2\pi}{3}$

get other angle by $2\pi - \frac{2\pi}{3} = \frac{4\pi}{3}$

③ look all sol's in $[0, 2\pi]$

Derivative of $\tan x$

$$f(x) = \tan(x) \overset{\text{def'n}}{=} \frac{\sin(x)}{\cos(x)}$$

$$f'(x) \underset{\text{quotient rule}}{=} \frac{\cos(x)\cdot\cos(x) - \sin(x)(-\sin(x))}{\cos^2(x)} = \frac{\overbrace{\cos^2(x) + \sin^2(x)}^{\text{trig ID} = 1}}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$= \left(\frac{1}{\cos(x)}\right)^2 = \sec^2(x)$$

$$\boxed{\frac{d}{dx}(\tan(x)) = \sec^2(x)}$$

$\tan = \frac{\sin}{\cos}$	$\cot = \frac{\cos}{\sin}$
$\sec = \frac{1}{\cos}$	
$\csc = \frac{1}{\sin}$	

$$\frac{d}{dx}(\sec(x)) = \frac{d}{dx}\left(\frac{1}{\cos(x)}\right) \underset{\text{quotient}}{=} \frac{-(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos(x)}$$

$$= \tan(x) \cdot \sec(x)$$

$$\boxed{\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)}$$

Same Idea $\Rightarrow$ $\boxed{\frac{d}{dx}(\csc(x)) = -\csc(x)\cdot\cot(x)}$

$$\frac{d}{dx}(\cot(x)) = \frac{d}{dx}\left(\frac{\cos(x)}{\sin(x)}\right) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\left(\frac{1}{\sin x}\right)^2$$

$$= -(\csc(x))^2$$

$$\boxed{\frac{d}{dx}(\cot(x)) = -\csc^2 x}$$

Motivation for Chain Rule

If person $\underline{a}$ walks twize as fast as person $\underline{b}$.

& person $\underline{b}$ walks 3 times as fast as person $\underline{c}$

How much faster does $\underline{a}$ walk than $\underline{c}$? 6 times faster.

$\underline{a}$: $\frac{da}{db} = 2$

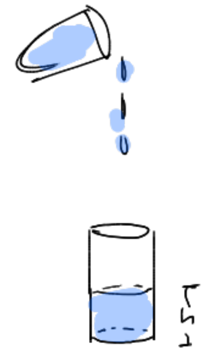
$\underline{b}$: $\frac{db}{dc} = 3$

Question: $\frac{da}{dc} = \underbrace{\frac{da}{db}}_{=2} \cdot \underbrace{\frac{db}{dc}}_{=3} = 6$

a = amount of water (volume) in receptacle.

time

h = height of water in receptacle



$\frac{dh}{da} = 3 \frac{\text{in}}{\text{amt}}$

$\frac{dh}{da}$ = change in height given change in volume $\approx 4.5 \frac{\text{inches}}{\text{amount}}$

$$\boxed{\frac{dh}{dt} = \frac{dh}{da} \cdot \frac{da}{dt}}$$

See Notes Page

Derivative Exercises

- Power Rule | Solutions
- Product Rule | Solutions
- Quotient Rule | Solutions
- Chain Rule - Powers | Solutions
- Chain Rule - Trig | Solutions
- Chain Rule - Exponential & Logs | Solutions
- Chain Rule - Inverse Trig | Solutions

start here

deriv. of outside

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

derivative of inside

Powers

$$f(x) = \left(\frac{1}{x} + 5x^2 + 1\right)^3$$

$$f'(x) = 3\left(\frac{1}{x} + 5x^2 + 1\right)^2 \cdot \left(-\frac{1}{x^2} + 10x\right)$$

stop

Idea:

$$\frac{d}{dx}\left(u^n\right) = n \cdot u^{n-1} \cdot \frac{du}{dx}$$

chain rule: identify inside & outside

(often between parenthesis

$$f(x) = \left(\frac{x+1}{x-1}\right)^2$$

think $u = \dfrac{x+1}{x-1}$ then $f = u^2$

$$\frac{df}{dx} = 2u' \cdot \frac{du}{dx}$$

$$f'(x) = 2\left(\frac{x+1}{x-1}\right)^1 \cdot \left(\frac{\overbrace{x-1-(x+1)}^{-2}}{(x-1)^2}\right)$$

quotient rule

$$= \boxed{\frac{-4(x+1)}{(x-1)^3}}$$

TRIG - CHAIN RULE.

$f(x) = \cos^2(x) = (\cos(x))^2 = u^2 \dashrightarrow 2u \cdot \frac{du}{dx}$

Key: figure out what the inside function is $(u = \cos x)$

$f'(x) = 2(\cos(x)) \cdot (-\sin x)$

$= -2\cos x \cdot \sin x$

$g(x) = \cos(2x)$ inside: $u = 2x \dashrightarrow \frac{d}{dx}(\cos(u)) = \overbrace{-\sin(u)}^{\text{deriv. of outside}} \cdot \frac{du}{dx}$

$g'(x) = -\sin(2x) \cdot 2 = -2\sin(2x)$