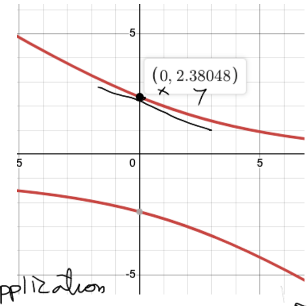
Thurs: Wk 6 Implicit Differentiation & Inverse Derivatives

1. warm-up
2. derivatives of inverse functions
   a. inverse trig
   b. ln(x)
   c. a^x



warm-up

Find $y' \left(= \frac{dy}{dx}\right)$ where $2xy + x + 3y^2 = 17$

$$\frac{d}{dx}\left(2xy + x + 3y^2\right) = \frac{d}{dx}(17)$$

$$\frac{d}{dx}(2xy) + \frac{d}{dx}(x) + \frac{d}{dx}(3y^2) = 0$$

product rule

$$2 \cdot y + 2x\frac{dy}{dx} + 1 + 6y \cdot \frac{dy}{dx} = 0$$

isolate $\frac{dy}{dx}$

$$\frac{dy}{dx}(2x + 6y) \overset{\text{factor}}{=} 2x\frac{dy}{dx} + 6y\frac{dy}{dx} = -1 - 2y \Longrightarrow \boxed{\frac{dy}{dx} = \frac{-1-2y}{2x+6y}}$$

slope of tan line

Application

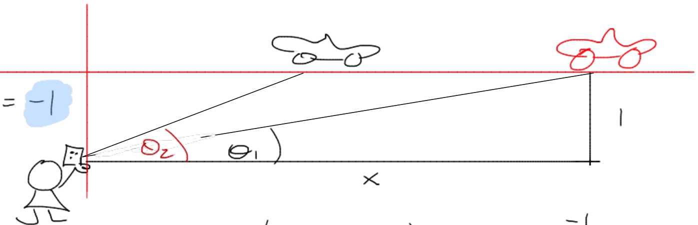
this eq'n has a graph

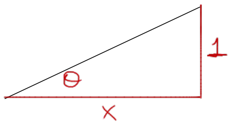
that is the

Derivative of arctan(x)

Question: (what's $\lim_{x\to 0} \frac{d}{dx}\left(\tan^{-1}\left(\frac{1}{x}\right)\right) = \lim_{x\to 0} \frac{-1}{x^2+1} = -1$

How does the angle change as the car gets closer? (as $x \to 0$)





Need relationships b/w x & θ (angle)?

since $\tan\theta = \frac{1}{x} \Rightarrow \tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{1}{x}\right)$

so $\theta = \tan^{-1}\left(\frac{1}{x}\right)$

how does θ change?

Need → $\frac{d\theta}{dx}$

$$\frac{d}{dx}\left(\tan^{-1}\left(\frac{1}{x}\right)\right) = \frac{-1}{x^2+1}$$

1st we need formula for deriv of arctan(x).

$$\frac{1}{1+\left(\frac{1}{x}\right)^2}\cdot\left(\frac{-1}{x^2}\right) = \frac{1}{\frac{x^2}{x^2}+\frac{1}{x^2}}\cdot\left(\frac{-1}{x^2}\right)$$

$$= \frac{1}{\frac{x^2+1}{x^2}}\cdot\left(\frac{-1}{x^2}\right) = \frac{x^2}{x^2+1}\cdot\left(\frac{-1}{x^2}\right)$$

$$= \frac{-1}{x^2+1}$$

Goal $\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$

1. write $y = \tan^{-1}(x)$, want $\frac{dy}{dx}$.

2. hit w/ inverse fcn:

$$\tan(y) = \tan\left(\tan^{-1}(x)\right) = x$$

$$\boxed{\tan(y) = x}$$

3. hit w/ $\frac{d}{dx}$:

$$\frac{d}{dx}(\tan(y)) = \frac{d}{dx}(x)$$

$$\sec^2(y)\cdot\frac{dy}{dx} = 1$$

5. relate $\sec^2(y)$ to x

mine: $\tan(y) = x$

so $\tan^2(y) = x^2$

also since $\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$

(*) $\tan^2 y + 1 = \sec^2 y$

4. Isolate $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{1}{\sec^2(y)} \overset{(*)}{=} \frac{1}{\tan^2(y)+1} = \frac{1}{x^2+1} = \frac{1}{1+x^2}$$

w/ Chain Rule:

$$\boxed{\frac{d}{dx}\left(\tan^{-1}(u)\right) = \frac{1}{1+u^2}\,\frac{du}{dx}}$$

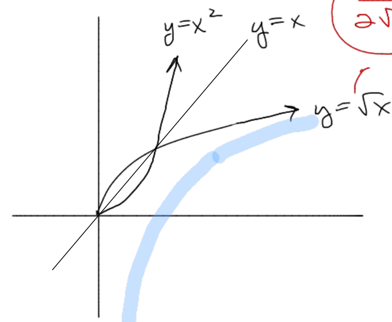
$$\frac{d}{dx}\left(\ln(x)\right)$$

① Set $y = \ln(x)$

② hit w/ inverse $\quad e^y = e^{\ln(x)} = x$

③ hit w/ $\frac{d}{dx}$

$$\left.\begin{array}{c}\frac{d}{dx}\left(e^y\right) = \frac{d}{dx}(x) \\ \parallel \\ e^y \cdot y' = 1\end{array}\right\} \quad y' = \frac{1}{e^y}$$



Faster $\frac{1}{2\sqrt{x}}$

④ relate $e^y \sim x$

$$y' = \frac{1}{e^y} = \frac{1}{x}$$

$$\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}$$

log growth is slower x^n for any n

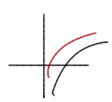
Chain Rule + ln :

$$\frac{d}{dx}\left(\ln(u)\right) = \frac{1}{u} \cdot \frac{du}{dx}$$

EX which grows faster: ln(x) or ln(2x) ?

$$\ln(2x) = \ln(2) + \ln(x)$$

$$\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}$$



$$\frac{d}{dx}\left(\ln(2x)\right) = \frac{1}{2x} \cdot 2 = \frac{1}{x}$$

$$\frac{d}{dx}(x^9e^x) = 9x^8e^x + x^9e^x = x^8e^x[9+x]$$

$$\frac{d^2}{dx^2} = [8x^7 \cdot e^x + x^8 \cdot e^x][9+x] + x^8e^x \cdot 1$$

distribute / factor