Derivatives: Wed Wk6

1. Warm-up:
    a. loose ends
    b. Higher Derivative notation
    c. Other trig derivatives
        i. cot, csc, sec
2. Implicit Diff
3. Chart

Warm-up:

$$\frac{d}{dx}\left(\cos^2(3\sqrt{x}+x)\right)$$

recall:
$\cos^2(x) = (\cos(x))^2$

"

$$\frac{d}{dx}\left(\left(\cos(3\sqrt{x}+x)\right)^2\right) = 2u \cdot \frac{du}{dx}$$

$$= 2(\cos(3\sqrt{x}+x)) \cdot (-\sin(u)) \cdot \frac{du}{dx}$$

$$= -2\cos(3\sqrt{x}+x) \cdot \sin(3\sqrt{x}+x) \cdot \left(\frac{3}{2}x^{-\frac{1}{2}}+1\right)$$

Higher Derivative Notation

Two ways to write derivatives.

$y = f(x)$

$y' = 1^{st}$ derivative (Newton)

or $\frac{dy}{dx}$ (Leibniz)

$y'' = 2^{nd}$ derivative

$\frac{d^2y}{dx^2}$ why? $\frac{d}{dx}\left(\frac{d}{dx}\right) = \frac{d^2}{dx^2}$

y''' etc $\frac{d^3y}{dx^3}$

Exercise

$y = \sin(x)$ $\qquad \frac{d^{100}}{dx^{100}}\left(\sin(x)\right) = \sin(x)$

5 $y' = \cos(x)$

6 $y'' = -\sin(x)$

7 $y''' = -\cos(x)$

8, 4 $\frac{d^4y}{dx^4} = -(-\sin(x)) = \sin(x)$

Other Trig Derivatives

$$\frac{d}{dx}\left(\sec(x)\right) = \frac{d}{dx}\left(\frac{1}{\cos(x)}\right) \overset{\text{quotient}}{=} \frac{\cos(x)\cdot 0 - (-\sin(x))}{\cos^2(x)} = \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)}\cdot\frac{1}{\cos(x)}$$

$$= \sec(x)\cdot\tan(x)$$

$$\frac{d}{dx}\left(\sec(x)\right) = \sec(x)\tan(x)$$

why should we care?

sec(x) <—> Mercator Projection

$$\frac{d}{dx}\left(\csc(x)\right) = \frac{d}{dx}\left(\frac{1}{\sin(x)}\right) = \frac{-\cos(x)}{\sin^2(x)} = -\frac{\cos(x)}{\sin(x)}\cdot\frac{1}{\sin(x)} = -\csc(x)\cdot\cot(x)$$

$$\frac{d}{dx}\left(\csc(x)\right) = -\csc(x)\cot(x)$$

$$-\sin^2 - \cos^2 = -(\sin^2 x + \cos^2 x)$$

$$\frac{d}{dx}\left(\cot(x)\right) = \frac{d}{dx}\left(\frac{\cos(x)}{\sin(x)}\right) = \frac{-\sin(x)\sin(x) - \cos(x)\cos(x)}{\sin^2(x)} = \frac{-1}{\sin^2(x)} = -\csc^2(x)$$

$$\frac{d}{dx}\left(\cot(x)\right) = -\csc^2(x)$$

So w/ the Chain Rule:

$$\left(\frac{d}{dx}\left(\sec(u)\right) = \sec(u)\tan(u)\cdot\frac{du}{dx}\right)$$

$$\frac{d}{dx}\left(\sec\left(\frac{1}{x}\right)\right) = \sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)\cdot\frac{-1}{x^2}$$

Implicit Differentiation

Not all functions are "explicit", like $y = \sqrt{x} + \frac{1}{x}$

(y is determined "explicitly" by x.

" y is alone "

For example

$$x^2 + y^2 = 1$$

here y is an implicit fcn of x,

we can apply $\frac{d}{dx}$ to both sides:

$$\frac{d}{dx}\left(x^2 + y^2\right) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}\left(y^2\right) = 0$$

b/c these variables match

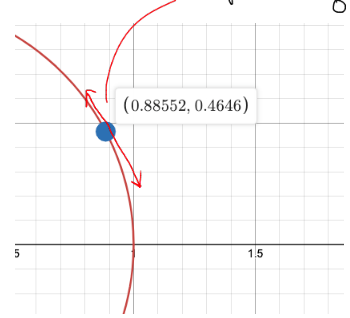
these don't match

$$2x \cdot \frac{dx}{dx}$$

$$2x + 2y\frac{dy}{dx} = 0$$

slope $= \frac{-0.8}{0.4} = \frac{-x}{y}$

≈ -2



Finally isolate y' (or $\frac{dy}{dx}$) $\Rightarrow$ $2y\frac{dy}{dx} = -2x$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}}$$