WED. WK 6

Exercises
(warm-up)

$f(x) = \cos(\sqrt{x})$

$f'(x) = -\sin(\sqrt{x}) \cdot \frac{1}{2} x^{-1/2}$
$= -\frac{1}{2} \cdot \sin(\sqrt{x}) \cdot x^{-1/2}$

$f(x) = \sin\left(\frac{1}{x}\right)$

$f'(x) = \cos\left(\frac{1}{x}\right) \cdot \frac{-1}{x^2}$

$f(x) = \tan\left(\frac{x+1}{1-\sqrt{x}}\right)$

$f'(x) = \sec^2\left(\frac{x+1}{1-\sqrt{x}}\right)\left[\frac{(1-\sqrt{x}) - (x+1)(-\frac{1}{2}x^{-\frac{1}{2}})}{(1-\sqrt{x})^2}\right]$

$f(x) = \cos(\sin(x))$

$f'(x) = -\sin(\sin(x)) \cdot \cos(x)$

$-\sin(\sin(x)) \cdot \cos(x)$

$f(x) = e^{\cos(\sqrt{x})}$

$f'(x) =$

$e^{\cos(\sqrt{x})} \cdot \left[-\frac{1}{2} \sin(\sqrt{x}) x^{-1/2}\right] = \frac{-1}{2\sqrt{x}} \cdot e^{\cos(\sqrt{x})} \cdot \sin(\sqrt{x})$

$\frac{d}{dx}(\tan(u))$
$=$
$\sec^2(u) \cdot \frac{du}{dx}$

$\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$

$\frac{d}{dx}(\cos(u)) = -\sin(u) \cdot \frac{du}{dx}$

$\frac{d}{dx}(\sin(u)) = \cos(u) \cdot \frac{du}{dx}$

Derivatives: Wed Wk6

1. Warm-up:
    a. loose ends
    b. Higher Derivative notation
    c. Other trig derivatives
        i. cot, csc, sec
2. Chart
3. Implicit Diff

Higher Derivative Notation

$y = \sin(x)$

9 5 $y' = \cos(x)$

10 6 $y'' = -\sin(x)$

11 7 $y''' = -\cos(x)$

12 8 $\frac{d^4y}{dx^4} = -(-\sin(x)) = \sin(x)$

$\frac{d^{100}y}{dx^{100}} = \sin(x)$

y' - Newton's notation for derivative of y.

y'' 2nd derivative

$\frac{dy}{dx}$ - Leibniz

$\frac{d^2y}{dx^2}$ - 2nd deriv

Other Trig Derivatives

quotient rule

$$\frac{d}{dx}\left(\sec(x)\right) = \frac{d}{dx}\left(\frac{1}{\cos(x)}\right) = \frac{\cos(x)\cdot 0 + 1\cdot\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)}\cdot\frac{1}{\cos(x)}$$

$$= \sec(x)\cdot\tan(x)$$

$$\frac{d}{dx}\left(\sec(x)\right) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\left(\csc(x)\right) = \frac{d}{dx}\left(\frac{1}{\sin(x)}\right) = \frac{0 - 1\cdot\cos(x)}{\sin^2(x)} = \frac{-\cos(x)}{\sin(x)}\cdot\frac{1}{\sin(x)} =$$

$$= -\csc(x)\cdot\cot(x)$$

$$\frac{d}{dx}\left(\csc(x)\right) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\left(\cot(x)\right) = \frac{d}{dx}\left(\frac{\cos(x)}{\sin(x)}\right) = \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = \frac{-1\,(\overbrace{\sin^2 \cancel{x} + \cos^2 \cancel{x}}^{=1})}{\sin^2} = \frac{-1}{\sin^2(x)} = -\csc^2(x)$$

$$\frac{d}{dx}\left(\cot(x)\right) = -\csc^2(x)$$

Ex

$$\frac{d}{dx}\left(\sec(\sqrt{x} + x)\right)$$

$$= \sec(\sqrt{x} + x)\tan(\sqrt{x} + x)\cdot\left[\frac{1}{2}x^{-\frac{1}{2}} + 1\right]$$

$$\frac{d}{dx}\sec(u) = \sec(u)\tan(u)\cdot\frac{du}{dx}$$

Implicit differentiation

First, consider

$$y = (\sqrt{x}+1)^2 + (e^x+1)^2$$

$\downarrow$ apply $\frac{d}{dx}$ to both sides

power power

$$\frac{d}{dx}(y) = \frac{d}{dx}\left((\sqrt{x}+1)^2 + (e^x+1)^2\right) = \frac{d}{dx}(\sqrt{x}+1)^2 + \frac{d}{dx}(e^x+1)^2$$

$$= (\sqrt{x}+1)\cdot(x^{-1/2}) + 2(e^x+1)\cdot e^x$$

Now

Find y' or $\frac{dy}{dx}$ as follows:

$$x^2 + y^2 = 1$$

$\downarrow$ apply $\frac{d}{dx}$

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(1) = 0$$

$$= \frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$$

$\downarrow$ don't match

shaded variables match

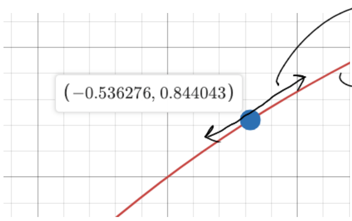
$$2x\cdot\underbrace{\frac{dx}{dx}}_{=1} + 2y\cdot\frac{dy}{dx} = 0$$

get $\frac{dy}{dx}$ by itself

$$2y\cdot\frac{dy}{dx} = -2x$$

$$\boxed{\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}}$$

$$\text{slope} = y' = \frac{dy}{dx} = \frac{-x}{y} = \frac{-(-0.5)}{0.8} \approx \frac{5}{8} \approx .6$$



$x^2+y^2=1$