



MA161 - Exam 2 - Guide

Name: _____

Find $f'(x)$.

power 1. $f(x) = \frac{5}{\sqrt{x}} = 5x^{-1/2}$, $f'(x) = \frac{5}{2}x^{-1/2}$

2. $f(x) = e^2 - e^x + x^e$
where is x?
 $f'(x) = 0 - e^x + e x^{e-1} = -e^x + e x^{e-1}$
x = base (\Rightarrow power rule still works for irrational exponents)
NONE

product 3. $f(x) = x^2 \sin x$
 $f'(x) = 2x \cdot \sin(x) - x^2 \cos(x)$

piece by piece 4. $f(x) = 4e^{-x} + \cos x - 9 \ln x$
 $f'(x) = -4e^{-x} - \sin(x) - \frac{9}{x}$

5. $f(x) = (2x^4 - 3e^{2x} + \tan x)^7$
 $f'(x) = 7(2x^4 - 3e^{2x} + \tan x)^6 (8x^3 - 6e^{2x} + \sec^2 x)$
power
power
tanh
copy
derivative of copy

6. (a) Find the slope of the tangent line to the graph of $y = \cos(x)$ at $(\frac{\pi}{4}, \frac{\sqrt{2}}{2})$.
plan: take deriv, plug in $x = \pi/4$

$$y' = -\sin(x)$$

$$y'(\pi/4) = \frac{-\sqrt{2}}{2} \approx -0.7$$

(b) Find the slope of the tangent line to the graph of $y = \cos^{-1}(x)$ at $(\frac{\sqrt{2}}{2}, \frac{\pi}{4})$.
same problem, same plan

$$y' = \frac{-1}{\sqrt{1-x^2}}, y'(\frac{\sqrt{2}}{2}) = \frac{-1}{\sqrt{1-(\frac{\sqrt{2}}{2})^2}} = \frac{-1}{\sqrt{1-\frac{2}{4}}} = \frac{-1}{\sqrt{1/2}} = \frac{-1}{(\frac{1}{\sqrt{2}})} = -\sqrt{2}$$

(c) Show that the answers to (a) and (b) are multiplicative inverses of each other.

(Hint: Show $\frac{1}{(a)} = (b)$ or $(a)(b) = 1$.)

$$\frac{-\sqrt{2}}{2} \cdot (-\sqrt{2}) = \frac{\sqrt{2} \cdot \sqrt{2}}{2} = \frac{2}{2} = 1$$



Forget
 deriv of $\sec(x)$ $\rightarrow \sec(x) = \frac{1}{\cos(x)}$ use quotient rule $\frac{\cos(x) \cdot 0 - 1 \cdot (-\sin(x))}{\cos^2(x)} = \frac{\sin(x)}{\cos^2(x)}$

$= \frac{\sin(x)}{\cos(x) \cdot \cos(x)}$

7. $f(x) = \sec^3 x = (\sec(x))^3$

power

$f'(x) = 3(\sec(x))^2 \cdot \sec(x) \tan(x)$

90% correct

$= 3 \sec^3(x) \tan(x)$

e^u

8. $f(x) = e^{\sec^3 x}$

$e^{\sec^3(x)} \cdot \underbrace{3 \cdot \sec^2(x) \cdot \tan(x)}_{\text{deriv of } \sec^3(x), \text{ see \#7}} = 3 \sec^3(x) \tan(x) e^{\sec^3(x)}$

9. $f(x) = \frac{x^3}{\cos x} = x^3 \cdot \frac{1}{\cos x} = x^3 \cdot \sec(x)$

quotient

product

$f'(x) = 3x^2 \cdot \sec(x) + x^3 \cdot \sec(x) \tan(x)$

$\frac{\cos(x) \cdot 3x^2 + x^3 \sin(x)}{\cos^2(x)}$

10. $f(x) = \sin^{-1}(x^3)$

arc-sin

$\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$

$\frac{1}{\sqrt{1-(x^3)^2}} \cdot 3x^2 =$

$\frac{3x^2}{\sqrt{1-x^6}}$

$$a^u \cdot \frac{du}{dx} \cdot \ln(a)$$

--- a^u x is in exponent

11. Use derivatives to determine which curve is steeper at $x=0$, $f(x) = 2^{3x+1}$ or $g(x) = 3^{2x+1}$.

steeper = slope w/ highest abs. value of derivative @ $x=0$

$$f'(x) = 2^{3x+1} \cdot 3 \cdot \ln(2), \quad f'(0) = 2 \cdot 3 \cdot \ln(2) = 6 \cdot \ln 2$$

$$g'(x) = 3^{2x+1} \cdot 2 \cdot \ln(3), \quad g'(0) = 3 \cdot 2 \cdot \ln(3) = 6 \cdot \ln 3$$

Ans: $g(x)$ is steeper

12. $f(x) = \sqrt{x^2 - 1}$

power = $(x^2 - 1)^{1/2}$

$$f'(x) = \frac{1}{2} (x^2 - 1)^{-1/2} \cdot (2x) = x (x^2 - 1)^{-1/2}$$

copy deriv of copy

$$\frac{x}{\sqrt{x^2 - 1}}$$

$\ln(u)$

13. $f(x) = \ln(\cos x)$

$$f'(x) = \frac{1}{\cos(x)} \cdot (-\sin(x)) = -\frac{\sin(x)}{\cos(x)} = -\tan(x)$$

Recall: $y - y_1 = m(x - x_1)$ eq'n of line
 plus x_1 into original
 gives (x_1, y_1) known #'s
 gives (x, y) variables

Ans: $y - \ln\left(\frac{\sqrt{2}}{2}\right) = -1\left(x - \frac{\pi}{4}\right)$

14. Find the equation of the tangent line to $f(x) = \ln(\cos x)$ at $x = \frac{\pi}{4}$.

Need two ingredients: point, slope

$x_1 = \text{given} = \frac{\pi}{4}$

$y_1 = \text{sub } x = \frac{\pi}{4} \text{ into original}$

(we see "tangent" \Rightarrow take deriv, plus in $x = \frac{\pi}{4}$)

slope $f'(x) = \frac{1}{\cos(x)} \cdot (-\sin(x)) = -\frac{\sin x}{\cos x} = -\tan x$
 need $\ln(u)$ derivative $\left(\frac{1}{u} \cdot \frac{du}{dx}\right)$

point
 $y_1 = \ln(\cos(\pi/4))$
 $= \ln\left(\frac{\sqrt{2}}{2}\right)$

$f'(\pi/4) = -\tan(\pi/4) = -1$

15. $f(x) = \csc(2x)$

$\csc(u)$

or $\frac{1}{\sin(2x)}$

w/ quotient

$$f'(x) = -\csc(2x) \cot(2x) \cdot 2$$

or

write $\csc(2x) = \frac{1}{\sin(2x)}$ & use quotient rule

$$\left(\frac{1}{\sin(2x)}\right)' = \frac{0 - \cos(2x) \cdot 2}{(\sin(2x))^2} = \frac{-2\cos(2x)}{(\sin(2x))^2}$$

16. $f(x) = x^{2x}$

hit w/ log algorithm

① $y = x^{2x}$

② $\ln(y) = \ln(x^{2x}) = 2x \cdot \ln(x)$

③ $\frac{1}{y} \cdot y' = 2 \cdot \ln(x) + 2x \left(\frac{1}{x}\right) = 2(\ln(x) + 1)$

④ $y' = 2(\ln(x) + 1) \cdot y$

⑤ $y' = 2(\ln(x) + 1) \cdot x^{2x}$

$$(x-h)^2 + (y-k)^2 = 1$$

17. Find all points (x, y) where the curve $(x-1)^2 + (y-1)^2 = 1$ has a horizontal tangent. Bonus: sketch a graph of the curve and its horizontal tangents.

plan: take deriv, set = 0, solve

$$\frac{d}{dx} [(x-1)^2 + (y-1)^2] = \frac{d}{dx} [1]$$

$$2(x-1) + 2(y-1) \cdot y' = 0$$

① solve for y' $2(y-1)y' = -2(x-1)$

②
$$y' = \frac{-2(x-1)}{2(y-1)} = \frac{-(x-1)}{y-1} = \frac{1-x}{y-1}$$

this is the deriv. \uparrow

③ Now $y' = 0$

$$\frac{1-x}{y-1} = 0$$

cross mult

$$1-x = 0 \cdot (y-1) = 0$$

$$\Rightarrow x = 1$$

To find all point (x, y) sub $x=1$ into original

$$(x-1)^2 + (y-1)^2 = 1 \quad \left| \begin{array}{l} \text{sq. root} \Rightarrow y-1 = \pm\sqrt{1} = \pm 1 \\ y-1 = 1, y = 2 \\ \text{or} \\ y-1 = -1, y = 0 \end{array} \right.$$

Bonus: circle, radius = 1, center = (1,1)

