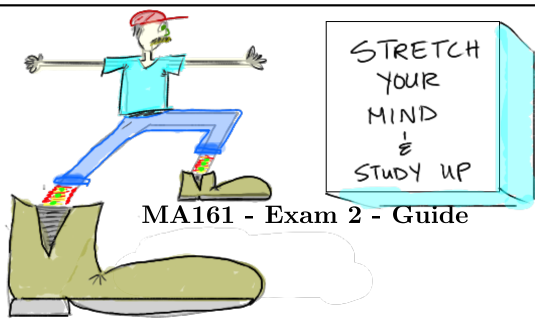

MA161 - Exam 2 - Guide **Name:** ____________________

Find $f'(x)$.

power 1. $f(x) = \dfrac{5}{\sqrt{x}} = 5x^{-1/2}$, $\boxed{f'(x) = -\frac{5}{2}x^{-3/2}}$

e^u x = base ($\Rightarrow$ power rule still works for irrational exponents)

where is x? 2. $f(x) = e^2 - e^x + x^e$ $f'(x) = 0 - e^x + ex^{e-1} = \boxed{-e^x + ex^{e-1}}$

NONE

product 3. $f(x) = x^2 \sin x$ $\boxed{f'(x) = 2x \cdot \sin(x) + x^2 \cos(x)}$

piece by piece 4. $f(x) = 4e^{-x} + \cos x - 9\ln x$ $\boxed{f'(x) = -4e^{-x} - \sin(x) - \frac{9}{x}}$

power 5. $f(x) = (2x^4 - 3e^{2x} + \tan x)^7$ e^u power   tan

$f'(x) = \boxed{7\underbrace{(2x^4 - 3e^{2x} + \tan x)}_{\text{copy}}{}^6\underbrace{(8x^3 - 6e^{2x} + \sec^2 x)}_{\text{derivative of copy}}}$

6. (a) Find the slope of the tangent line to the graph of $y = \cos(x)$ at $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$.

plan: take deriv., plug in $x = \pi/4$

$y' = -\sin(x)$

$y'(\pi/4) = \boxed{-\frac{\sqrt{2}}{2}} \approx -.7$

(b) Find the slope of the tangent line to the graph of $y = \cos^{-1}(x)$ at $\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$.

same problem, same plan

$y' = \dfrac{-1}{\sqrt{1-x^2}}$, $y'\left(\frac{\sqrt{2}}{2}\right) = \dfrac{-1}{\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}} = \dfrac{-1}{\sqrt{1-\frac{2}{4}}} = \dfrac{-1}{\sqrt{1/2}} = \dfrac{-1}{\left(\frac{1}{\sqrt{2}}\right)} = \boxed{-\sqrt{2}}$

(c) Show that the answers to (a) and (b) are multiplicative inverses of each other.
(Hint: Show $\frac{1}{(a)}$ = (b) or (a)(b) = 1.)

$\boxed{-\frac{\sqrt{2}}{2} \cdot (-\sqrt{2}) = \frac{\sqrt{2}\cdot\sqrt{2}}{2} = \frac{2}{2} = 1}$ ☺

Forget deriv of $sec(x)$??? $\rightarrow$ $sec(x) = \frac{1}{cos(x)}$ use quotient rule $\Big\}$ $\frac{cos x \cdot 0 - 1\cdot(-sin(x))}{cos^2(x)} = \frac{sin x}{cos^2 x}$ $= \frac{sin}{cos} \cdot \frac{1}{cos}$

7. $f(x) = \sec^3 x$ $= (sec(x))^3$

power

$f'(x) = 3(sec(x))^2 \cdot sec(x) tan(x)$ 90% correct

$= \boxed{3 sec^3(x) tan(x)}$

e^u

8. $f(x) = e^{\sec^3 x}$

$e^{sec^3(x)} \cdot \underbrace{3 \cdot sec^3(x) \cdot tan x}_{\text{deriv of } sec^3(x) \text{, see \#7}} = \boxed{3 sec^3(x) tan(x) e^{sec^3(x)}}$

9. $f(x) = \frac{x^3}{\cos x}$ $= x^3 \cdot sec(x)$

$x^3 \cdot \frac{1}{cos} =$

quotient or product

$f'(x) = \boxed{3x^2 \cdot sec(x) + x^3 \cdot sec(x) tan(x)}$

$\boxed{\frac{cos(x) \cdot 3x^2 + x^3 sin(x)}{cos^2(x)}}$ or

10. $f(x) = \sin^{-1}(x^3)$

arc-sin

$\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$

$\frac{1}{\sqrt{1-(x^3)^2}} \cdot 3x^2 = \boxed{\frac{3x^2}{\sqrt{1-x^6}}}$

$a^u \cdot \frac{du}{dx} \cdot \ln(a)$ ← - - - a^u x is in exponent

11. Use derivatives to determine which curve is steeper at $x = 0$, $f(x) = 2^{3x+1}$ or $g(x) = 3^{2x+1}$.

steeper = slope w/ highest abs. value of derivative @ $x=0$

$f'(x) = 2^{3x+1} \cdot 3 \cdot \ln(2)$, $f'(0) = 2 \cdot 3 \cdot \ln(2) = 6 \cdot \ln 2$

$g'(x) = 3^{2x+1} \cdot 2 \cdot \ln(3)$, $g'(0) = 3 \cdot 2 \cdot \ln(3) = 6 \cdot \ln 3$ bigger

ANS: g(x) is steeper

12. $f(x) = \sqrt{x^2 - 1}$

power $= (x^2-1)^{1/2}$

$f'(x) = \frac{1}{2}(x^2-1)^{-1/2} \cdot (2x) = x(x^2-1)^{-1/2} = \boxed{\frac{x}{\sqrt{x^2-1}}}$

copy deriv of copy

ln(u)

13. $f(x) = \ln(\cos x)$

$f'(x) = \frac{1}{\cos(x)} \cdot (-\sin(x)) = \frac{-\sin(x)}{\cos(x)} = \boxed{-\tan(x)}$

Recall: $y - y_1 = m(x - x_1)$ eq'n of line (m = slope)

(x_1, y_1) known #'s (given)

(x, y) variables

plug x_1 into original

ANS: $y - \ln\left(\frac{\sqrt{2}}{2}\right) = -1\left(x - \frac{\pi}{4}\right)$

14. Find the equation of the tangent line to $f(x) = \ln(\cos x)$ at $x = \frac{\pi}{4}$.

Need two ingredients: point, slope (we see "tangent" ⇒ take deriv, plug in $x = \pi/4$)

x_1 = given $= \frac{\pi}{4}$

y_1 = sub $x = \frac{\pi}{4}$ into original

slope $f'(x) = \frac{1}{\cos(x)} \cdot (-\sin(x)) = \frac{-\sin x}{\cos x} = -\tan x$

need ln(u) derivative $\left(\frac{1}{u} \cdot \frac{du}{dx}\right)$

$f'(\pi/4) = -\tan(\pi/4) = -1$

point

$y_1 = \ln(\cos(\pi/4))$

$= \ln\left(\frac{\sqrt{2}}{2}\right)$

15. $f(x) = \csc(2x)$

$\csc(u)$

or

$\dfrac{1}{\sin(2x)}$ w/ quotient

$f'(x) = \boxed{-\csc(2x)\cot(2x)\cdot 2}$

$\boxed{\text{or}}$

write $\csc(2x) = \dfrac{1}{\sin(2x)}$ & use quotient rule

$\left(\dfrac{1}{\sin(2x)}\right)' = \dfrac{0-\cos(2x)\cdot 2}{(\sin(2x))^2} = \boxed{\dfrac{-2\cos(2x)}{(\sin(2x))^2}}$

16. $f(x) = x^{2x}$

hit w/ log algorithm

1. $y = x^{2x}$
2. $\ln(y) = \ln(x^{2x}) = 2x \cdot \ln(x)$
3. $\frac{1}{y}\cdot y' = 2\cdot\ln(x) + 2x\left(\frac{1}{x}\right) = 2(\ln(x)+1)$
4. $y' = 2(\ln(x)+1)\cdot y$
5. $y' = 2(\ln(x)+1)\cdot x^{2x}$

$(x-h)^2 + (y-k)^2 = 1$

17. Find all points (x, y) where the curve $(x-1)^2 + (y-1)^2 = 1$ has a $\underbrace{\text{horizontal tangent}}$. Bonus: sketch a graph of the curve and its horizontal tangents.

plan: , take deriv, I
, set = 0
, solve

$\frac{d}{dx}\left[(x-1)^2 + (y-1)^2\right] = \frac{d}{dx}[1]$

don't match ↑ get y'

$2(x-1) + 2(y-1)\cdot y' = 0$

I solve for y'

1. $2(y-1)\,y' = -2(x-1)$

2. $y' = \dfrac{-2(x-1)}{2(y-1)} = \dfrac{-(x-1)}{y-1} = \dfrac{1-x}{y-1}$

this is the deriv.

II Now $y' = 0$

$\dfrac{1-x}{y-1} = 0$ cross mult. $1 - x = 0\cdot(y-1) = 0$

so $x = 1$

To find all point (x,y) sub $x=1$ into original

$(x-1)^2 + (y-1)^2 = 1$

$x=1 \Rightarrow 0 + (y-1)^2 = 1$

sq. root $\Rightarrow y - 1 = \pm\sqrt{1} = \pm 1$

$y-1 = 1$, $y = 2$

or

$y - 1 = -1$, $y = 0$

Bonus: circle, radius $= 1$, center $= (1,1)$