

Monday - MA161

1. Exam - Thursday

product

If $f(t) = (t^2 + 3t + 2)(2t^{-2} + 5t^{-3})$, find $f'(t)$.

Answer:

$$f'(t) = (2t + 3)(2t^{-2} + 5t^{-3}) + (t^2 + 3t + 2)(-4t^{-3} - 15t^{-4})$$

stop

$$\text{Find } y' = \frac{dx}{dy}$$

$$\sin(x) + \cos(y) = \sin(x) \cdot \cos(y)$$

$$\frac{d}{dx} \left(\sin(x) + \cos(y) \right) = \frac{d}{dx} \left(\overset{1^{\text{st}}}{\sin(x)} \cdot \overset{2^{\text{nd}}}{\cos(y)} \right)$$

+ product

Hint:

$$\frac{d}{dx} (\sin(x)) = \cos(x)$$

$$\frac{d}{dx} (\sin(3x)) = \cos(3x) \cdot 3$$

$$\cos(x) + \left(-\sin(y) \cdot \frac{dy}{dx} \right) = \frac{d}{dx} \left[\overset{1^{\text{st}}}{\sin(x)} \cdot \overset{2^{\text{nd}}}{\cos(y)} \right]$$

$$= \cos(x) \cdot \cos(y) + \sin(x) (-\sin(y)) \frac{dy}{dx}$$

CHART

$$\frac{d}{dx} (\sin(u)) = \cos(u) \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{d}{dx} (\sin(x)) = \cos(x) \cdot \left(\frac{dx}{dx} \right) = 1$$

$$= \cos(x)$$

$$\cos(x) - y' \cdot \sin(y) = \cos(x) \cos(y) - y' \sin(x) \sin(y)$$

Now use algebra to isolate y' .

$$\cos(x) - \cos(x) \cos(y) = y' \sin(y) - y' \sin(x) \sin(y)$$



$$= y' \underbrace{(\sin(y) - \sin(x) \sin(y))}_{\downarrow}$$

$$\frac{\cos(x) - \cos(x) \cos(y)}{\sin(y) - \sin(x) \sin(y)} = y' \quad \leftarrow$$

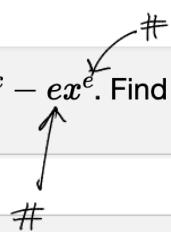


$$\frac{\cos(x)[1 - \cos(y)]}{\sin(y)[1 - \sin(x)]} = y'$$

#3/

Suppose that $f(x) = 9e^x - ex^e$. Find $f'(3)$.

$$f'(3) =$$



$$f'(x) = 9e^x - e \cdot ex^{e-1}$$

$$f'(x) = 9e^x - e^2 x^{e-1}$$

$$f'(3) = 9e^3 - e^2 \cdot 3^{e-1}$$

$$e^x \xrightarrow{d/dx} e^x$$

$$x^2 \rightarrow 2x$$

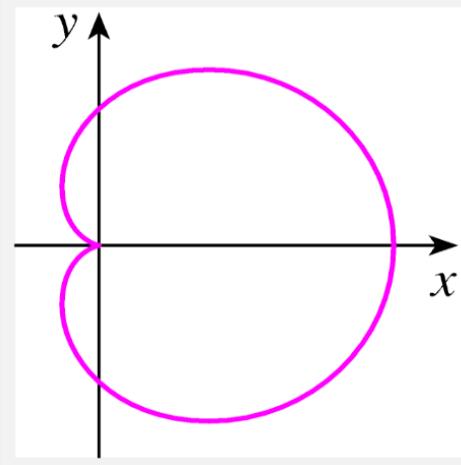
$$x^\pi \rightarrow \pi x^{\pi-1}$$

$$x^{\frac{1}{2}} \rightarrow \frac{1}{2} x^{\frac{1}{2}-1}$$

Power

Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$$x^2 + y^2 = (2x^2 + 2y^2 - x)^2, (0, 1/2)$$



Plan:

① Hit both sides w/ $\frac{d}{dx}$, use chain rule¹

get an equation like

$$2x + 2y \cdot y' = 2(2x^2 + 2y^2 - x)(4x + 4y \cdot y' - 1)$$

② ^{First} plus in $x=0$, solve for y' .
 $y' = 1/2$

$$\text{eq. } 0 + y' = (17 - x)(3y')$$

this is the
slope

③ Point: $(0, \frac{1}{2})$

$$\Rightarrow y - y_1 = m(x - x_1)$$

Differentiate $y = \sqrt{1-x^2} \sin^{-1} x$.

$$y' =$$

Recall: $\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

Note: product

$$\frac{1}{2} (1-x^2)^{-\frac{1}{2}} (-2x) \cdot \sin^{-1} x + \sqrt{1-x^2} \cdot \frac{1}{\sqrt{1-x^2}}$$

$$= -x (1-x^2)^{-\frac{1}{2}} \cdot \sin^{-1} x + 1$$

$$\sqrt{1-x^2} = (1-x^2)^{\frac{1}{2}}$$

$$\downarrow \quad \begin{array}{c} -\frac{1}{2} \\ \frac{1}{2}-1 \end{array}$$
$$\frac{1}{2} (1-x^2) \cdot (-2x)$$

6. (a) Find the slope of the tangent line to the graph of $y = \cos(x)$ at $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$.

$$y' = -\sin x$$

$$y'\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} = \text{slope}$$

- (b) Find the slope of the tangent line to the graph of $y = \cos^{-1}(x)$ at $\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$.

$$y' = \frac{-1}{\sqrt{1-x^2}}, \quad y'\left(\frac{\sqrt{2}}{2}\right) = \frac{-1}{\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}} = -\frac{1}{\frac{1}{\sqrt{2}}} = -\sqrt{2}$$

- (c) Show that the answers to (a) and (b) are multiplicative inverses of each other.

(Hint: Show $\frac{1}{(a)} = (b)$ or $(a)(b) = 1$.)

$$-\frac{\sqrt{2}}{2} \cdot -\sqrt{2} = \frac{(-\sqrt{2})(-\sqrt{2})}{2} = \frac{2}{2} = 1$$