Wk 7 — Mon

HW 3.9.9

$y = 7^{x^3}$

$\left(\overset{\text{variable}}{(\text{const})}\right) \implies a^u$

$$\boxed{\frac{d}{dx}(a^u) = a^u \frac{du}{dx} \ln(a)}$$

(a constant, u variable)

$y' = 7^{x^3} \cdot 3x^2 \cdot \ln(7)$

3.9.6

$$y(x) = \ln\left(\underbrace{2(\ln(x))^9}_{u}\right) = \ln(u)$$

$$y'(x) = \frac{1}{u} \cdot \frac{du}{dx} = \frac{1}{2(\ln(x))^9} \cdot 18 \cdot (\ln(x))^8 \cdot \frac{1}{x} = \frac{9}{x \cdot \ln(x)}$$

$$\frac{du}{dx} = \frac{d}{dx}\left(2 \cdot (\ln x)^9\right)$$

$$= 2 \cdot 9 \cdot (\ln x)^8 \cdot \frac{1}{x}$$

— Derivative Chart —

Function	Derivative
$\ln(u)$	$\frac{1}{u} \cdot \frac{du}{dx}$
u^n	$n \cdot u^{n-1} \frac{du}{dx}$
e^u	$e^u \frac{du}{dx}$
$\sin(u)$	$\cos(u) \frac{du}{dx}$
$\cos(u)$	$-\sin(u) \frac{du}{dx}$
$\tan(u)$	$\sec^2(u) \frac{du}{dx}$
$\sec(u)$	$\sec(u)\tan(u) \frac{du}{dx}$
$\csc(u)$	$-\csc(u)\cot(u) \frac{du}{dx}$
$\cot(u)$	$-\csc^2(u) \frac{du}{dx}$

Function	Derivative
a^u	$a^u \cdot \frac{du}{dx} \cdot \ln(a)$
$\tan^{-1}(u)$	$\frac{1}{1+u^2} \frac{du}{dx}$
$\sin^{-1}(u)$	$\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$
$\sec^{-1}(u)$	$\frac{1}{\lvert u \rvert \sqrt{u^2-1}} \frac{du}{dx}$
$\cot^{-1}(u)$	$\frac{-1}{1+u^2} \frac{du}{dx}$
$\cos^{-1}(u)$	$\frac{-1}{\sqrt{1-u^2}} \frac{du}{dx}$
$\csc^{-1}(u)$	$\frac{-1}{\lvert u \rvert \sqrt{u^2-1}} \frac{du}{dx}$

Ex

$$\frac{d}{dx}\left(\sin^{-1}(\sin(x))\right) \underset{\text{chain rule}}{\overset{\text{via}}{=}} \frac{1}{\sqrt{1-(\sin(x))^2}} \cdot \cos(x) = \frac{\cos(x)}{\sqrt{\underbrace{1-\sin^2 x}_{\cos^2 x}}} = \frac{\cos x}{\sqrt{\cos^2 x}} = 1$$

$$\frac{d}{dx}\left(\sin^{-1}(u)\right) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

here $u = \sin(x)$

so $\frac{du}{dx} = \cos(x)$

sub

$$\frac{d}{dx}(x) = 1$$

$$\boxed{\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}}$$

(inverse function idea)

① set $y = \sin^{-1}(x)$, want $\frac{dy}{dx}$

② hit w/ inverse fcn $\boxed{\sin(y) = x}$

③ hit w/ $\frac{d}{dx}$

$$\frac{d}{dx}\left(\sin(y)\right) = \frac{d}{dx}(x)$$

$$\cos(y)\cdot\frac{dy}{dx} = 1$$

④ isolate $\frac{dy}{dx} = \frac{1}{\cos(y)}$

⑤ relate $\cos(y)$ to x ... via mine step ②
get x back

square ② $\Rightarrow$ $\sin^2(y) = x^2$

since $\sin^2 + \cos^2 = 1$

$$\cos^2(y) = 1 - \sin^2(y)$$

$$\cos(y) = \pm\sqrt{1-\sin^2 y}$$

⑥ $$\frac{dy}{dx} = \frac{1}{\pm\sqrt{1-\sin^2(y)}} = \frac{1}{\sqrt{1-\sin^2(y)}}$$

b/c $\sin^{-1}(x)$ is increasing

⑦ $$dy/dx = \frac{1}{\sqrt{1-x^2}}$$

Ex

$$y = \ln\left(\sqrt{\frac{x^2+1}{x^2-1}}\right)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{\frac{x^2+1}{x^2-1}}} \cdot \frac{1}{2}\left(\frac{x^2+1}{x^2-1}\right)^{-1/2} \cdot \frac{(x^2-1)2x - (x^2+1)(2x)}{(x^2-1)^2}$$

quotient rule

better

$$y = \ln\left(\frac{x^2+1}{x^2-1}\right)^{1/2} = \frac{1}{2}\ln\left(\frac{x^2+1}{x^2-1}\right) = \frac{1}{2}\left[\ln(x^2+1) - \ln(x^2-1)\right]$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{2x}{x^2+1} - \frac{2x}{x^2-1}\right]$$