Mon. Wk 7

Today: Chart

Applications of Deriv.

warm-up:

$f(x) = 9\sin(x) - \dfrac{10}{\cos(x)}$

$f'(x) = 9\cos(x) - \dfrac{\cos(x)\cdot 0 - \overbrace{10(-\sin(x))}^{10\sin(x)}}{\cos^2(x)}$

$= 9\cos(x) - \dfrac{10\sin(x)}{\cos^2(x)}$

OR

$f(x) = 9\sin(x) - 10\sec(x)$

$f'(x) = 9\cos(x) - 10\sec(x)\tan(x)$

# — Derivative Chart —

| Functions | Derivatives $\frac{d}{dx}$ |
|---|---|
| $u^n$ | $n \cdot u^{n-1} \cdot \frac{du}{dx}$ |
| $e^u$ | $e^u \cdot \frac{du}{dx}$ |
| $\ln(u)$ | $\frac{1}{u} \cdot \frac{du}{dx}$ |
| $\sin(u)$ | $\cos(u) \cdot \frac{du}{dx}$ |
| $\cos(u)$ | $-\sin(u)\frac{du}{dx}$ |
| $\tan(u)$ | $\sec^2(u)\frac{du}{dx}$ |
| $\sec(u)$ | $\sec(u)\tan(u)\frac{du}{dx}$ |
| $\csc(u)$ | $-\csc(u)\cot(u)\frac{du}{dx}$ |
| $\cot(u)$ | $-\csc^2(u)\frac{du}{dx}$ |

| Functions | Derivatives |
|---|---|
| $a^u$ | $a^u \ln(a) \cdot \frac{du}{dx}$ |
| $\tan^{-1}(u)$ | $\frac{1}{1+u^2} \cdot \frac{du}{dx}$ |
| $\sin^{-1}(u)$ | $\frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$ |
| $\sec^{-1}(u)$ | $\frac{1}{\lvert u\rvert\sqrt{u^2-1}} \frac{du}{dx}$ |
| $\cot^{-1}(u)$ | $\rightarrow \frac{-1}{1+u^2} \frac{du}{dx}$ |
| $\cos^{-1}(u)$ | $\frac{-1}{\sqrt{1-u^2}} \frac{du}{dx}$ |
| $\csc^{-1}(u)$ | $\rightarrow \frac{-1}{\lvert u\rvert\sqrt{u^2-1}} \frac{du}{dx}$ |

Find $y'(x)$

[redacted] $y(x) = \ln\left(2\underbrace{(\ln(x))}_{u}^{9}\right)$.

$$y'(x) = \frac{1}{2(\ln(x))^9} \cdot 18(\ln(x))^8 \cdot \frac{1}{x}$$

$$= \frac{9}{x \cdot \ln(x)}$$

Recall:

$$\ln(u) \xrightarrow{d/dx} \frac{1}{u} \cdot \frac{du}{dx}$$

$$2 \cdot u^9 \xrightarrow{d/dx} 9 \cdot 2 \cdot u^8 \cdot \frac{du}{dx}$$

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Again: (notice the impact of the 9)

$$y = \ln(\underbrace{2 \cdot \ln(x)})$$

$$y' = \frac{1}{2 \cdot \ln(x)} \cdot 2 \cdot \frac{1}{x} = \frac{1}{x \cdot \ln(x)}$$

$$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$$

⓪ Use inverse function idea

① set $y = \sin^{-1}(x)$, want $\frac{dy}{dx}$

② hit w/ inverse: $\boxed{\sin(y) = x}$

$\left[x' \to 1 \cdot x^0 \cdot \frac{dx}{dx} = 1 \cdot 1 \cdot 1 = 1\right]$

③ hit w/ $\frac{d}{dx}$

$\cos(y) \cdot \frac{dy}{dx} = 1$

$(\cos^2 + \sin^2 = 1$

$\cos^2 = 1 - \sin^2$

$\sqrt{\cos^2} = \sqrt{1-\sin^2}$

④ Isolate $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\cos(y)}$

$\cos(y) = \pm\sqrt{1-\sin^2(y)}$

(since $\sin^{-1}(x)$ is increasing fn $\cos(y) =$ positive

⑤ "get x back" (mine step ②)

square $\Rightarrow \sin^2(y) = x^2$

$$\frac{dy}{dx} = \frac{1}{\pm\sqrt{1-\sin^2(y)}} \overset{\sin^{-1} \text{ is increasing} \Rightarrow +}{=} \frac{1}{\sqrt{1-\sin^2(y)}} \overset{⑥}{=} \frac{1}{\sqrt{1-x^2}}$$