Tue - wk 7

3.8.2

1. hit w/ $\frac{d}{dx}$, use implicit diff
2. isolate $\frac{dy}{dx}$, set = 0, solve

**Question 2 of 9**

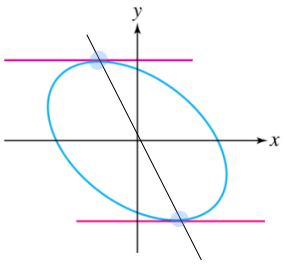
Find all points on the graph of $3x^2 + 6y^2 + 3xy = 64$ where the tangent line is horizontal.

product

$\frac{dy}{dx}$ = slope = 0

$\frac{d}{dx} = 6x + 12y\cdot\frac{dy}{dx} + 3\left(y + x\cdot\frac{dy}{dx}\right) = 0$

$3y + 3x\frac{dy}{dx}$




(1)

$12y\frac{dy}{dx} + 3x\frac{dy}{dx} = -3y - 6x$

$\frac{dy}{dx}(\text{nnn}) = -3y - 6x$

$\frac{dy}{dx} = \frac{-3y - 6x}{12y + 3x}$

(Give your answer as a comma-separated list of points in the form (*,*). Express numbers in exact form. Use symbolic notation and fractions where needed.)

(2) $\frac{dy}{dx} = \frac{-3y - 6x}{12y + 3x} = 0 \rightsquigarrow -3y - 6x = 0$

$\Rightarrow 3y = -6x$

$y = -2x$

coordinates of the point(s): $(-8/\sqrt{21}, 16/\sqrt{21})$, $(8/\sqrt{21}, -16/\sqrt{21})$

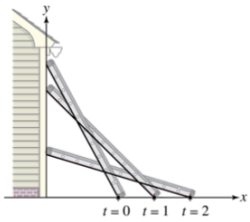
(3) To find the exact coords: combine ← w/ given!

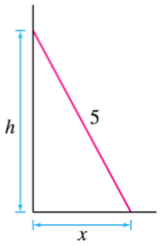
$3x^2 + 6(-2x)^2 + 3x(-2x) = 64$ quadratic (degree 2) poly

$21x^2 = 3x^2 + 24x^2 - 6x^2 = 64 \Rightarrow x = \pm\sqrt{64/21} = \pm\frac{8}{\sqrt{21}}$

Today: 3-10 (follow text)

A 5-m ladder leans against a wall. Assume the bottom slides away from the wall at a rate of 0.9 m/s.




The variable $h$ is the height of the ladder's top at time $t$, and $x$ is the distance from the wall to the ladder's bottom.

Find the velocity of the top of the ladder at $t = 2$ s if the bottom is 1.5 m from the wall at $t = 0$ s.

(Use decimal notation. Give your answer to three decimal places.)

derivative w.r.t. time (m/s)

① Read twice

② Find goal: "?" velocity of top @ time $t=2$ → $\left.\frac{dh}{dt}\right|_{t=2}$

③ Translate to math + extract data. (look @ units)

$\Rightarrow \boxed{0.9 \frac{m}{s} = \frac{dx}{dt}}$

④ Relate variables $x, h$: (often uses geometry)

$\boxed{x^2 + h^2 = 5^2}$

⑤ Differentiate ($\frac{d}{dt}$) in order to "relate the rates"

$2x \cdot \frac{dx}{dt} + 2h \cdot \frac{dh}{dt} = 0$

⑥ Isolate $\frac{dh}{dt}$

$2x\frac{dh}{dt} = -2x\frac{dx}{dt} \Rightarrow \boxed{\frac{dh}{dt} = \frac{-x}{h} \cdot \frac{dx}{dt}}$

Formula (at all time) for velocity of top (we want it when $t = 2$)

⑦ Find $x|_{t=2}$ & $h|_{t=2}$

From #: $t = 0 \leftrightarrow x = 1.5$

From given: $dx/dt = 0.9$

$\boxed{x = 0.9t + 1.5}$

⑧ $x(2) = 0.9(2) + 1.5 = 3.3$ m

$h = \sqrt{25 - 3.3^2} \approx 4$

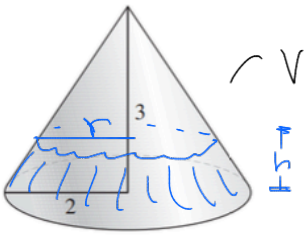
⑨ Plug in values into $\frac{dh}{dt} \approx -\frac{3.3}{3.75}(0.8) = -.79 \frac{m}{s}$

Now ———————————————————

$\left.\frac{dh}{dt}\right|_{h=0.01}$ → $x = \sqrt{25 - 0.01} \approx$ $\Rightarrow \frac{dh}{dt} \approx \frac{-5}{0.01}(0.9) \approx -450 \frac{m}{s}$ ouch!

(How fast is top ladder moving when it's almost flat?

A conical tank has height 3 m and radius 2 m at the base. Water flows in at a rate of 2 $m^3$/min.




$V = \frac{1}{3}\pi R^2 H$

see text

V = Total Vol − Top Empty Cone

How fast is the water level rising when the level is 1 m and when the level is 1.9 m?

(Use decimal notation. Give your answers to four decimal places.)