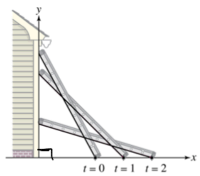
Thu - wk 7

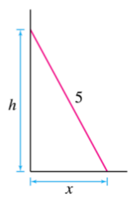
Questions:

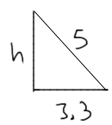
Falling ladder

A 5-m ladder leans against a wall. Assume the bottom slides away from the wall at a rate of 0.9 m/s.

$\frac{dx}{dt} = 0.9$







The variable h is the height of the ladder's top at time t, and x is the distance from the wall to the ladder's bottom.

Find the velocity of the top of the ladder at $t = 2$ s if the bottom is 1.5 m from the wall at $t = 0$ s. $\quad x\big|_{t=0} = 1.5$

h

goal: $\frac{dh}{dt}$ @ $t = 2$ or $= \frac{dh}{dt}\Big|_{t=2}$

1. Read Twice. look for ? or what's goal.
2. Label knowns.
3. Strategy: "Find a velocity" ≡ take derivative
   - get a formula (equation) that we differentiate so we can "relate the rates"
4. Equation (often get from geometry) that relates x & h

   $x^2 + h^2 = 5^2$ (pyth. thm)
5. Relate the rates via $\frac{d}{dt}$

   $2x \cdot \frac{dx}{dt} + 2h\frac{dh}{dt} = 0$
6. clean & isolate $\frac{dh}{dt}$: $\boxed{\frac{dh}{dt} = -\frac{x}{h}\frac{dx}{dt}}$
7. Plug in current state of $x, h, \frac{dx}{dt}$ when $t = 2$

   To get formula for x: combine $\frac{dx}{dt} = 0.9$ & $x\big|_{t=0} = 1.5$ to get $x = 0.9t + 1.5$

   know constant deriv. $\Rightarrow f(t) = mt + b$

   $x\big|_{t=2} = 0.9(2) + 1.5 = 1.8 + 1.5 = 3.3$ m

   use P.T. to find $h = \sqrt{5^2 - 3.3^2} \approx \sqrt{22.7} \approx 4.5$
8. $\frac{dh}{dt} \approx -\frac{3.3}{4.5}(0.9)$

Next: Compare values of $\frac{dh}{dt}$ when (a) $h = 0.5$
(b) $h = 0.1$
(c) $h = 0.01$ (just above floor)

(a) $\frac{dh}{dt} = \frac{-x}{h}(0.9)$ $\quad x\big|_{h=0.5} = \sqrt{25 - 0.5^2} \approx 4.9 \Rightarrow \frac{dh}{dt} \approx -\frac{4.9}{0.5}(0.9)$

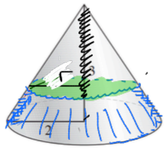
(b) $\frac{dh}{dt} = \frac{-4.99}{0.1}(0.9) \approx -45\frac{m}{s}$ $\quad x\big|_{h=0.1} = \sqrt{25 - 0.1^2} \approx 4.99$

(c) $\frac{dh}{dt} = \frac{-4.995}{0.01}(0.9) \approx -450\frac{m}{s}$ $\quad x\big|_{h=0.01} = \sqrt{25 - 0.01^2} \approx 4.995$

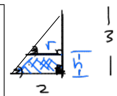
Points faster as it falls!

$H=3 \quad R=2$

A conical tank has height 3 m and radius 2 m at the base. Water flows in at a rate of 2 m^3/min. units! $\frac{m^3}{min} = \frac{vol}{time} = \boxed{\frac{dV}{dt} = 2}$



$\boxed{\text{goal: } \left.\frac{dh}{dt}\right|_{h=1}}$



As water rises, h changes, let r = radius of top surface of water

deriv.

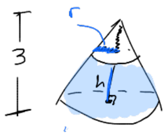
How fast is the water level rising when the level is 1 m and when the level is 1.9 m?

(Use decimal notation. Give your answers to four decimal places.)

Relate h, r, V : key: water volume is difference b/w total volume & vacuum (empty) space above

$\left.\frac{dh}{dt}\right|_{h=1} =$ ____ m/min

$\left.\frac{dh}{dt}\right|_{h=1.9} =$ ____ m/min



— Empty space —
$V = \frac{1}{3}\pi r^2 (3-h)$

total space

$V = \frac{1}{3}\pi \cdot 2^2 \cdot 3$
$= 4\pi$

$\boxed{V = \text{vol. of water} = 4\pi - \frac{1}{3}\pi r^2 (3-h)}$

similar △'s! triangles w/ same angles → ratio of corresp sides is same

(before we hit w/ $\frac{d}{dx}$, get only 1 variable (w/ geometry) relate h & r

$\frac{3}{2} = \frac{3-h}{r} \therefore r = \frac{2}{3}(3-h)$

update: $V = 4\pi - \frac{\pi}{3}\left(\frac{4}{9}(3-h)^2\right)(3-h) = \boxed{4\pi - \frac{4\pi}{27}(3-h)^3 = V}$

hit w/ $\frac{d}{dt}$: $\frac{dV}{dt} = -\frac{4\pi}{27} \cdot 3\,(3-h)^2 \cdot \left(-\frac{dh}{dt}\right)$ ← this is a relation of rates

base isn't t

know $\frac{dV}{dt} = 2$, $h = 1$ or $h = 1.9$ } plus in, isolate $\frac{dh}{dt}$