Derivative Chart - Draft #1

$u = f(x)$

Functions	Derivatives
$u^n \quad (n \in \mathbb{R})$	$n \cdot u^{n-1} \cdot \frac{du}{dx}$
$\sin(u)$	$\cos(u) \cdot \frac{du}{dx}$
$\cos(u)$	$-\sin(u) \frac{du}{dx}$
$\tan(u)$	$\sec^2(u) \frac{du}{dx}$
$\csc(u)$	$-\csc(u)\cot(u) \frac{du}{dx}$
$\sec(u)$	$\sec(u)\tan(u) \frac{du}{dx}$
$\cot(u)$	$-\csc^2(u) \frac{du}{dx}$
e^u	$e^u \cdot \frac{du}{dx}$

Functions	Derivatives
$\ln(u)$	$\frac{1}{u} \cdot \frac{du}{dx}$
$\sin^{-1}(u)$	$\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$
a^u	$a^u \cdot \frac{du}{dx} \cdot \ln(a)$

$$e^{2x} \xrightarrow{d/dx} e^{2x} \cdot 2$$

$$5^{2x} \xrightarrow{d/dx} 5^{2x}\, 2 \cdot \ln(5)$$

B/c $\overset{\text{set}}{y = a^x}$ w/ $a \in \mathbb{R}^+$

(1) hit w/ ln: $\ln(y) = \ln(a^x) \overset{\text{log property}}{=} \underbrace{x \cdot \underbrace{\ln(a)}_{\#}}$

eg $x \cdot 5$ or $x \cdot 3$ — $3x \rightarrow$

derivative

$\downarrow$ 5 $\downarrow$ 3

(2) $\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(x \cdot \ln(a))$

$$\frac{1}{y} \cdot y' = \ln(a)$$

$$y' = y \cdot \ln(a) = a^x \cdot \ln(a)$$

$$\frac{d}{dx}(a^x) = a^x \cdot \ln(a)$$

$\downarrow$

$$\boxed{\frac{d}{dx}(a^u) = a^u \cdot \frac{du}{dx} \cdot \ln(a)}$$

MA161 - Exam 2 - Guide **Name:** ______________________

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Show your work!

Find $f'(x)$.

1. $f(x) = \frac{5}{\sqrt{x}} = 5x^{-1/2}$, $-\frac{5}{2}x^{-3/2}$

power

2. $f(x) = e^2 - e^x + x^e$ $e^2 \to 0$, x^e: power $\boxed{f'(x) = -e^x + ex^{e-1}}$

careful w/ where x is

3. $f(x) = x^2 \sin x$

product

4. $f(x) = 4e^{-x} + \cos x - 9\ln x$

$\uparrow$ chain

5. $f(x) = (2x^4 - 3e^{2x} + \tan x)^7$ $\approx u^7 \longrightarrow 7u^6 \frac{du}{dx}$

power

$7(2x^4 - 3e^{2x} + \tan x)^6 \cdot (8x^3 - 6e^{2x} + \sec^2 x)$

6. (a) Find the slope of the tangent line to the graph of $y = \cos(x)$ at $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$.

(b) Find the slope of the tangent line to the graph of $y = \cos^{-1}(x)$ at $\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$.

(c) Show that the answers to (a) and (b) are multiplicative inverses of each other. (Hint: Show $\frac{1}{(a)} =$ (b) or (a)(b) = 1.)

Related:

$f(x) = \frac{x + x^2}{\sqrt{x}} = x^{\frac{1}{2}} + x^{3/2}$

algebra 1st

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{3}{2}x^{\frac{1}{2}}$

or

$f(x) = \frac{x(1+2x) - (x+x^2)\frac{1}{2}x^{-1/2}}{x}$

DIY Chart

u^n	$nu^{n-1}\frac{du}{dx}$
$\sin(u)$	$\cos(u)\frac{du}{dx}$
$\cos(u)$	$-\sin(u)\frac{du}{dx}$
$\csc(u)$	$-\csc(u)\cot(u)\frac{du}{dx}$
$\cot(u)$	$-\csc^2(u)\frac{du}{dx}$
$\tan(u)$	$\sec^2(u)\frac{du}{dx}$
e^u	$e^u\frac{du}{dx}$

$\ln(\text{blah})$	$\frac{1}{\text{blah}} \cdot \frac{d(\text{blah})}{dx}$
$\sin^{-1}(u)$	$\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$
a^u	$a^u\frac{du}{dx}\ln(a)$

7. $f(x) = \sec^3 x$

8. $f(x) = e^{\sec^3 x}$

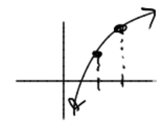
9. $f(x) = \dfrac{x^3}{\cos x}$

10. $f(x) = \sin^{-1}(x^3)$

$2^{3x+1} = 2^{3x+1} \cdot 3 \cdot \ln(2)$ $\qquad a^{u} \longrightarrow a^{u} \cdot \frac{du}{dx} \cdot \ln(a)$

11. Use derivatives to determine which curve is steeper at $x = 0$, $f(x) = 2^{3x+1}$ or $g(x) = 3^{2x+1}$.

steeper = most extreme slope



$\updownarrow$ largest derivative

$f'(x) = 2^{3x+1} \cdot 3 \cdot \ln(2)$, $f'(0) = 2^{3\cdot 0+1} \cdot 3 \cdot \ln(2) = 6 \cdot \ln(2)$

$g'(x) = 3^{2x+1} \cdot 2 \cdot \ln(3)$, $g'(0) = 3^{2\cdot 0+1} \cdot 2 \cdot \ln(3) = 6 \cdot \ln(3)$

steeper curve = $g(x)$

12. $f(x) = \sqrt{x^2 - 1}$

power

13. $f(x) = \ln(\cos x)$

$f'(x) = \frac{1}{\cos(x)} \cdot (-\sin(x)) = \frac{-\sin(x)}{\cos(x)} = -\tan(x)$

14. Find the equation of the tangent line to $f(x) = \ln(\cos x)$ at $x = \frac{\pi}{4}$.

slope = $f'(\pi/4)$

point = $\left(\frac{\pi}{4}, f\left(\frac{\pi}{4}\right)\right)$

15. $f(x) = \csc(2x)$

16. $f(x) = x^{2x}$

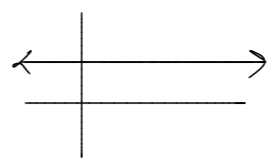
17. Find all points (x, y) where the curve $(x-1)^2 + (y-1)^2 = 1$ has a horizontal tangent. Bonus: sketch a graph of the curve and its horizontal tangents.

goal: $\{(x_1, y_1), (x_2, y_2)\}$

↳ take derivative y'
set = 0
solve

① $\frac{d}{dx}$ to both sides $\frac{d}{dx}\left((x-1)^2 + (y-1)^2\right) = \underbrace{\frac{d}{dx}(1)}_{=0}$

power rule



② $2(x-1)^1 \cdot 1 + 2(y-1)^1 \cdot y' = 0$ $y' = dy/dx$

derivative of $x-1$ derivative of $y-1$

③ isolate y' $y' = \frac{-2(x-1)}{2(y-1)} = \frac{-(x-1)}{(y-1)} = 0$ (take deriv), now set = 0

④ $\frac{-(x-1)}{y-1} = 0 = \frac{0}{1} \Longrightarrow -(x-1) = 0$ so $x = 1$

cross-mult

$\boxed{(1,0), (1,2)}$

⑤ Plug $x=1$ into original $\Rightarrow (x-1)^2 + (y-1)^2 = 1 \Rightarrow (y-1)^2 = 1$, $y - 1 = \pm 1$ ‖ $y = 1+1 = 2$ or $y = 1-1 = 0$

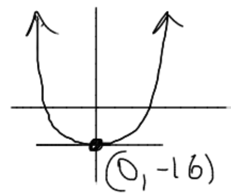
Horizontal tangent Problem

(repl slope of tan = 1)

Find where the graph has a <u>horiz. tangent</u>.

① take deriv.
② set = 0

$f(x) = x^4 - 16$



$f'(x) = 4x^3$

set = 0 $\quad 4x^3 = 0$

solve $\quad x = 0 \quad$ the x-value that makes the tangent horizontal

get the corresponding y.

$f(0) = 0^4 - 16 = -16$

$\boxed{(0, -16)}$

$f'(x) = 4x^3 = 1 \Rightarrow x = \sqrt[3]{1/4}$, get y by plugging into original

$y = f(\sqrt[3]{1/4}) = (1/4)^{4/3} - 16 \approx -12$

$\approx (\sqrt[3]{1/4}, -12)$