Wk 7 - Wed

Text

3-10 · related rates
4-1 linearization
4-2 optimization

Homework

pushed
open
thru
exam (Thursday)

3-10 HW

10 questions

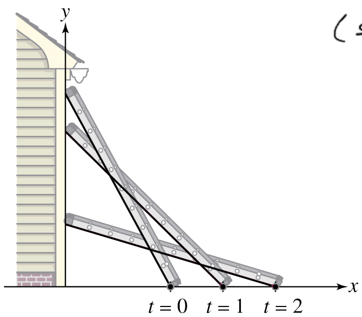
Course Info

Instructor Name

Student Name

Question 1 of 10

A 5-m ladder leans against a wall. Assume the bottom slides away from the wall at a rate of 0.9 m/s.



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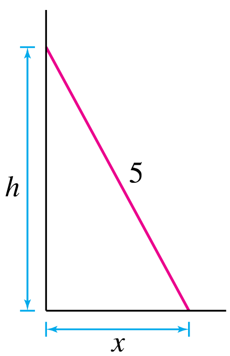


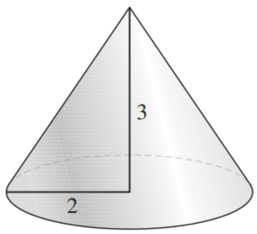
The variable h is the height of the ladder's top at time t, and x is the distance from the wall to the ladder's bottom.

Find the velocity of the top of the ladder at $t = 2$ s if the bottom is 1.5 m from the wall at $t = 0$ s.

(Use decimal notation. Give your answer to three decimal places.)

$\left.\frac{dh}{dt}\right|_{t=2}$ m/s

A conical tank has height 3 m and radius 2 m at the base. Water flows in at a rate of 2 m^3/min.



(see text)

How fast is the water level rising when the level is 1 m and when the level is 1.9 m?

(Use decimal notation. Give your answers to four decimal places.)

$\left.\frac{dh}{dt}\right|_{h=1} =$ ____ m/min

$\left.\frac{dh}{dt}\right|_{h=1.9} =$ ____ m/min

Question 3 of 10

A road perpendicular to a highway leads to a farmhouse located $a = 1.4$ km away.

An automobile travels past the farmhouse at a speed of $v = 86$ km/h. How fast is the distance between the automobile and the farmhouse increasing when the automobile is 3.2 km past the intersection of the highway and the road?

Let l denote the distance between the automobile and the farmhouse, and let s denote the distance past the intersection of the highway and the road.

1 Knowns

$a = 1.4$ km

$v = 86 \frac{km}{h} = \frac{dx}{dt}$

2 Set

x = dist b/w car & intersection

D = dist b/w car & house

t = time

3 Goal

$\left.\frac{dD}{dt}\right|_{x=3.2}$

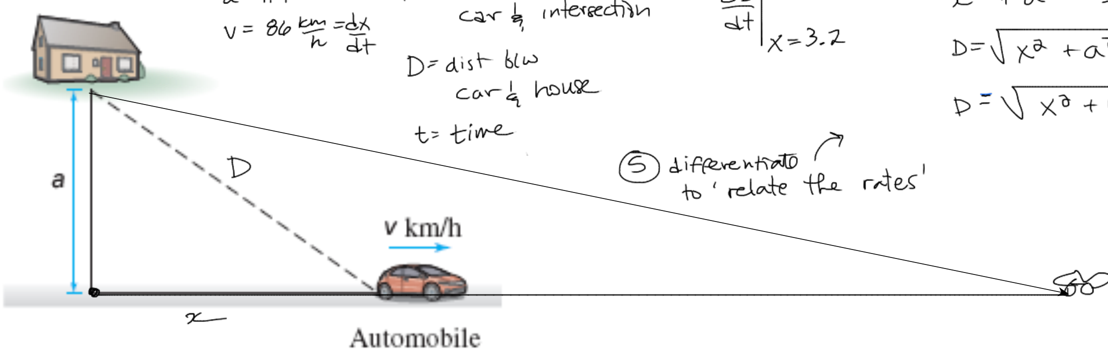
4 Relate variables (use geometry)

$x^2 + a^2 = D^2$

$D = \sqrt{x^2 + a^2}$

$D = \sqrt{x^2 + 1.4^2}$

5 differentiate to 'relate the rates'



(Use decimal notation. Give your answer to three decimal places.)

speed of l at given s-value: km/h

Question 4 of 10

The base x of the right triangle in the figure increases at a rate of 6 cm/s, while the height remains constant at $h = 18$ cm.

① Knowns

$t =$ time (sec)

$\frac{dx}{dt} = 6$

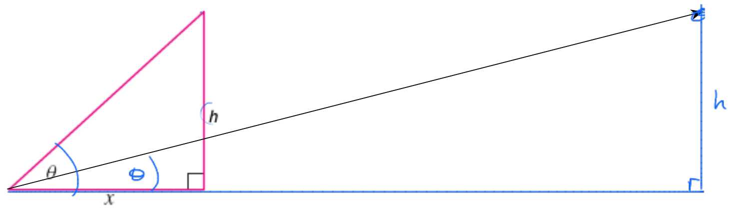
$h = 18$

② goal:

$\left.\frac{d\theta}{dt}\right|_{x=27}$

$\frac{1}{x} \xrightarrow{d/dx}$

$x^{-1} \rightarrow$



How fast is the angle θ changing with respect to time t in seconds when $x = 27$?

(Enter an exact answer. Use symbolic notation and fractions where needed.)

③ Relate $\tan\theta = \frac{h}{x}$

④ Differentiate

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{h}{x}\right) = \frac{d}{dt}\left(\frac{18}{x}\right) = \frac{d}{dt}\left(18x^{-1}\right)$$

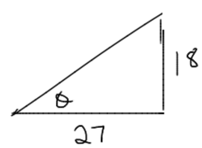
$\left.\frac{d\theta}{dt}\right|_{x=27} =$ ____________ rad/s

$$\sec^2\theta \cdot \frac{d\theta}{dt} = -18x^{-2} \cdot \frac{dx}{dt}$$

⑤ goal? isolate $d\theta/dt$

$$\frac{d\theta}{dt} = \frac{-18}{x^2} \cdot \frac{dx}{dt} \cdot \left(\frac{1}{\sec\theta}\right)^2 = \frac{-18}{x^2} \cdot \cos^2\theta \cdot \frac{dx}{dt}$$

what is $\cos^2\theta$?



Question 5 of 10

Assume that the radius r of a sphere is expanding at a rate of 10 cm/min. The volume of a sphere is $V = \frac{4}{3}\pi r^3$ and its surface area is $4\pi r^2$. Determine the rate at which the surface area is changing with respect to time when $r = 40$ cm.

(Use symbolic notation and fractions where needed.)

goal: $\left.\frac{dS}{dt}\right|_{r=40}$

known: $S = 4\pi r^2$

Idea

Chain Rule

$4\pi(x^3+1)^2$

$\downarrow d/dx$

$8\pi(x^3+1)\cdot 3x^2$

$\frac{dA}{dt} =$ ______ cm^2/min

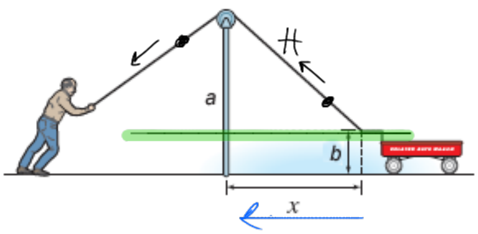
③ differentiate & sub-in as appropriate

$$\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt}$$

Question 7 of 10

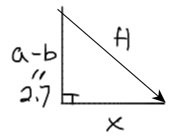
Placido pulls a rope attached to a wagon through a pulley at a rate of q m/s.

↳ #



(1) Knowns

$q = \frac{dH}{dt}$



(2) goal: $\frac{dx}{dt}$

Let $a = 3.2$ m and $b = 0.5$ m.

Find a formula for the speed of the wagon in terms of q and variable x in the figure.

deriv. of position (length)

$\frac{dx}{dt} =$ ______

(3) Relate: $H^2 = 2.7^2 + x^2$

(4) $\frac{d}{dt}$: $\cancel{2} \cdot H \cdot \frac{dH}{dt} = \cancel{2}x \cdot \frac{dx}{dt}$

(5) goal: $\frac{dx}{dt} = \frac{H}{x} \cdot \frac{dH}{dt}$

(6) Relate H to x: use (3) $H = \sqrt{2.7^2 + x^2}$

Finally, $\frac{dx}{dt} = \frac{\sqrt{2.7^2 + x^2}}{x} \cdot q$