Wed. Wk 7

Concepts / Chapters

3-10 : Related Rates

4-1 : Linearization 

4-2 : Optimization 

Question 3 of 10

A road perpendicular to a highway leads to a farmhouse located $a = 1.4$ km away. derv

An automobile travels past the farmhouse at a speed of $v = 86$ km/h. How fast is the distance between the automobile and the farmhouse increasing when the automobile is 3.2 km past the intersection of the highway and the road?

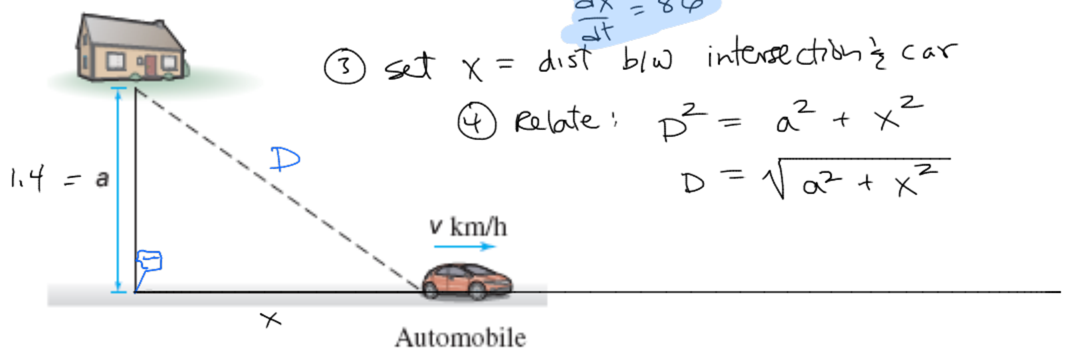
Let l denote the distance between the automobile and the farmhouse, and let s denote the distance past the intersection of the highway and the road.

① know: car speed $= 86$ km/h, $\frac{dx}{dt} = 86$ ② goal: $\frac{dD}{dt}$

③ set $x =$ dist b/w intersection & car

④ Relate: $D^2 = a^2 + x^2$

$D = \sqrt{a^2 + x^2}$



(Use decimal notation. Give your answer to three decimal places.)

speed of l at given s-value: ______ km/h

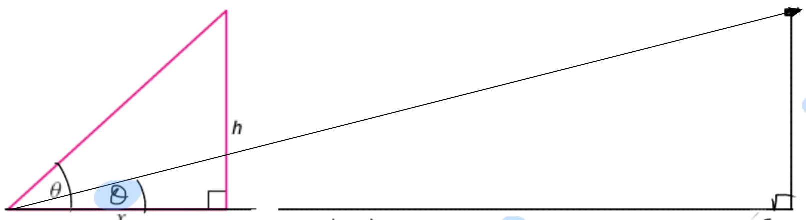
Question 4 of 10

The base x of the right triangle in the figure increases at a rate of 6 cm/s, while the height remains constant at $h = 18$ cm.

① goal: $\left.\frac{d\theta}{dt}\right|_{x=27}$

② know: $\frac{dx}{dt} = 6$

$h = 18$



derivative of θ wrt time x

③ Relate variables! (geometry!)

$\tan\theta = \frac{h}{x}$

How fast is the angle θ changing with respect to time t in seconds when $x = 27$?

(Enter an exact answer. Use symbolic notation and fractions where needed.)

$\left.\frac{d\theta}{dt}\right|_{x=27} =$ ______ rad/s

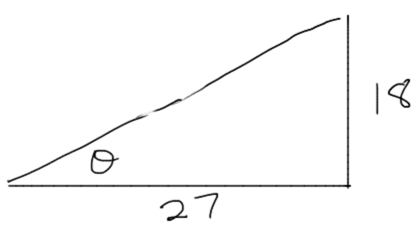
④ Now differentiate (hit w/ $\frac{d}{dt}$)

$$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{h}{x}\right) = \frac{d}{dt}\left(\frac{18}{x}\right) = \frac{d}{dt}\left(18x^{-1}\right)$$

$$\sec^2\theta \frac{d\theta}{dt} = -18x^{-2} \cdot \frac{dx}{dt}$$

⑤ Recall goal: Isolate $\frac{d\theta}{dt} = \frac{-18}{x^2} \cdot \frac{dx}{dt} \cdot \frac{1}{\sec^2\theta} = \frac{-18\cos^2\theta}{x^2} \cdot \frac{dx}{dt}$

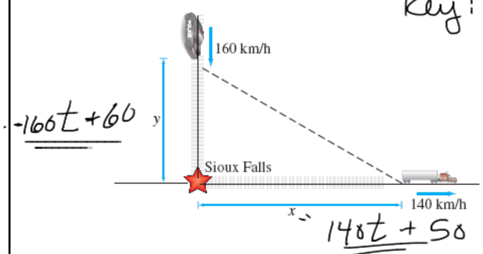
Now that we have a formula for $\frac{d\theta}{dt}$, set $x = 27$ and the corresponding θ.



what's $\cos^2\theta$?

Question 9 of 10

A police car traveling south toward Sioux Falls, Iowa, at 160 km/h pursues a truck traveling east away from Sioux Falls at 140 km/h.



Key: y is decreasing $\Rightarrow \frac{dy}{dt} < 0$

x is increasing $\Rightarrow \frac{dx}{dt} > 0$

At time $t = 0$, the police car is 60 km north and the truck is 50 km east of Sioux Falls.

Calculate the rate at which the distance between the vehicles is changing at $t = 5$ minutes.

(Use decimal notation. Give your answer to three decimal places.)

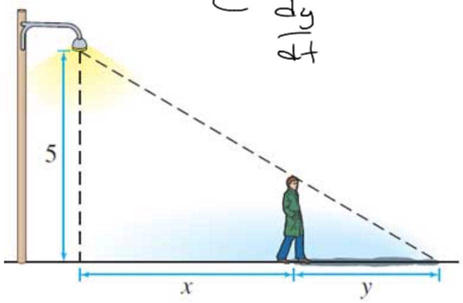
rate: ______ km/h

Question 10 of 10

A man of height 1.6 meters walks away from a 5-meter lamppost at a speed of 1.1 m/s. Find the rate at which his shadow is increasing in length.

Find: $\frac{dy}{dt}$

Key: Similar Δ's

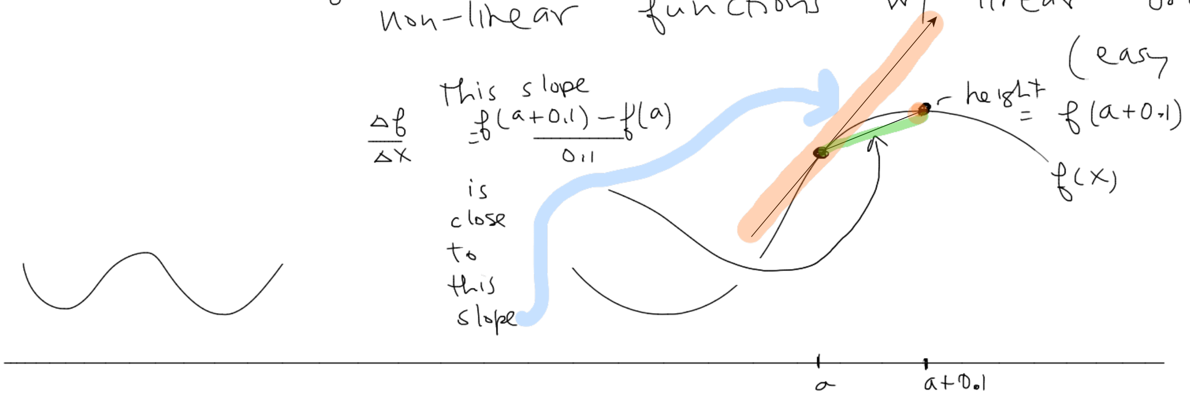


(Use decimal notation. Give your answer to three decimal places.)

rate: m/s

4.1 / Linearization

Idea: Real-world models are non-linear (hard to predict)
but if the underlying function is smooth
(always in Calc I)
you can approximate
non-linear functions w/ linear ones
(easy to compute)



Linearization: Just the equation of the tangent line

Ex $f(x) = \sqrt{x}$, Linearization of f @ $x=4$. $L(x) = \frac{1}{4}x + 1$

① Eqn of tan. line @ $x=4$

slope: $\frac{1}{2}x^{-1/2}\Big|_{x=4} = \frac{1}{2}\cdot\frac{1}{\sqrt{4}} = \frac{1}{4}$

pt $(4,2)$

$y = \frac{1}{4}x + b$

$2 = \frac{1}{4}(4) + b$

$1 = b$