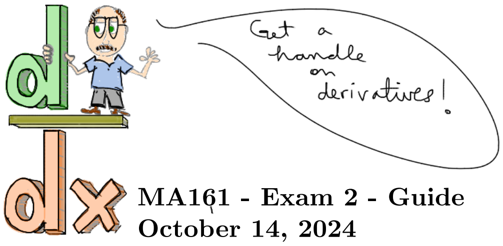


MA161 - Exam 2 - Guide **Name:** ____________________

October 14, 2024

Show your work!

Find $f'(x)$.

1. $f(x) = \ln 4 + e^4$

   $f'(x) = 0$

2. $f(x) = -\pi x$

   $-\pi$

3. $f(x) = 4e^{-x} + \tan x + 16 \ln x$

   $-4e^{-x} + \sec^2(x) + \frac{16}{x}$

4. $f(x) = (4x^7 - 5e^{4x} + \cos x)^8$ watch parenthesis!

   $8(4x^7 - 5e^{4x} + \cos(x))^7 \cdot (28x^6 - 20e^{4x} - \sin(x))$

5. $f(x) = (\ln x)^6$

   $\frac{6(\ln x)^5}{x}$

6. $f(x) = \frac{x + 3x^2 + 4\sqrt{x}}{\sqrt{x}}$ Hint: Algebra first.

   $= x^{\frac{1}{2}} + 3x^{\frac{3}{2}} + 4 \rightarrow \frac{1}{2}x^{-1/2} + \frac{9}{2}x^{1/2}$

7. $f(x) = x^6 \sec x$

   $6x^5 \cdot \sec(x) + x^6 \sec(x)\tan(x)$

8. $f(x) = 6^{\ln x}$

   $6^{\ln x} \cdot \frac{1}{x} \cdot \ln 6 = \frac{6^{\ln x} \cdot \ln 6}{x}$

9. $f(x) = e^{\cos^2 x}$

$$-e^{\cos^2 x} \cdot (2\cos x) \sin x$$

10. $f(x) = \ln(\cos^2 x)$

$$\frac{-2\cos x \sin x}{\cos^2 x} = -2 \frac{\sin x}{\cos x} = -2\tan x$$

11. $f(x) = x^5 \sin^{-1} x$

$$5x^4 \cdot \sin^{-1} x + x^5 \cdot \frac{1}{\sqrt{1-x^2}}$$

12. $f(x) = \cos^{-1}(x^4)$

$$\frac{-4x^3}{\sqrt{1-x^8}}$$

(12.5) At a time t seconds after it is thrown up in the air, a tomato is at a height (in meters) of

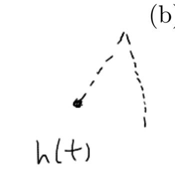
$$f(t) = -4.9t^2 + 55t + 1m.$$

(a) Find the $\underbrace{\text{instantaneous velocity}}$ of the tomato at time $t = 1$ (include units!)

$$f'(t) = -9.8t + 55$$

$$f'(1) = -9.8(1) + 55 = 45.2 \, \frac{m}{s}$$

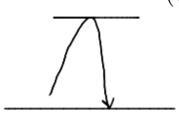
(b) Is the tomato going up or coming down at time $t = 4$? (justify your answer)



$\text{sign}(f'(4)) \Rightarrow$ up since $f'(4) > 0$

$$f'(4) = -9.8(4) + 55 \approx -44 + 55 > 0$$

(c) How high does the tomato go?



global max

set $= 0$

$f'(t) = -9.8t + 55 = 0$ $\quad t = \frac{55}{9.8} \approx 5.62$ sec

$f(5.62) = -4.9(5.62)^2 + 55(5.62) + 1 = $ 155 meters

13. $f(x) = \sqrt{x^2 - 1}$

$$\frac{1}{2}(x^2-1)^{-\frac{1}{2}} \cdot 2x = \frac{x}{\sqrt{x^2-1}}$$

14. $f(x) = \ln(\ln x)$

$$\frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \cdot \ln x}$$

15. $f(x) = \frac{1-e^x}{1+e^x}$

quotient

$$\frac{(1+e^x)(-e^x) - (1-e^x)(e^x)}{(1+e^x)^2}$$

$$= \frac{-e^x - e^{2x} - e^x + e^{2x}}{(1+e^x)^2} = \frac{-2e^x}{(1+e^x)^2}$$

16. $f(x) = \frac{1}{\sin(2x)}$

quotient

or

$\frac{1}{\sin} = \csc$

$$\longrightarrow \frac{-1 \cdot \cos(2x) \cdot 2}{\sin^2(2x)} = -\frac{2\cos(2x)}{\sin^2(2x)} = -2\cot(2x) \cdot \csc(2x)$$

$$\csc(2x) \longrightarrow \underline{-2\csc(2x)\cot(2x)}$$

17. $f(x) = x^{7x}$

set $y = x^{7x}$, find y'

$$\ln(y) = \ln(x^{7x}) = 7x \cdot \ln x$$

$$\downarrow \, d/dx$$

$$\frac{1}{y} \cdot y' = 7 \cdot \ln x + 7x \cdot \frac{1}{x} = 7\ln x + 7 = 7(\ln x + 1)$$

$$y' = 7(\ln x + 1)y = \boxed{7(\ln x + 1)x^{7x}}$$

see following pages

18. A boat leaves Marquette at 3:00 PM and travels due north at a speed of 10 m/h. Another boat has been heading west at 15 m/h and reaches Marquette at 5:00 PM. At what time were the boats closest together?

19. Find all points (x, y) where the curve $(x-1)^2 + (y-1)^2 = 1$ has a horizontal tangent.

20. There are two tangent lines to the curve $x^2 + xy = 1$ that have slope equal to -2. Find equations for them.

21. Find the equation of the tangent line to the graph of $y = (x^2+1)\sin x$ at $x = 0$. Use the linearization to approximate $(1.05)^{10}$.

22. Suppose the volume of a spherical balloon increases at a rate of $24\frac{cm^3}{sec}$. Find the rate that its diameter is increasing when the diameter is $3cm$.

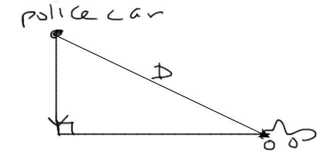
23. Find the absolute maximum and absolute minimum of the function on the indicated interval.

$$f(x) = \frac{x^4}{4} - 2x^2 + 1, \;\; [-3, 1]$$

18. A boat leaves Marquette at 3:00 PM and travels due north at a speed of 10 m/h. Another boat has been heading west at 15 m/h and reaches Marquette at 5:00 PM. At what time were the boats closest together? ⇒ Minimize D, local Min!

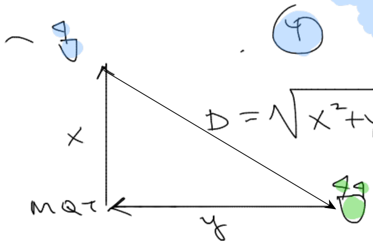
similar to:

police car



Plan: Find D'
Set D' = 0
Solve

4



$D = \sqrt{x^2+y^2} = \sqrt{(10t)^2 + (-15t+30)^2}$

1 let x = dist(, MQT) @ t = # hours since 3:00 pm

2 $\frac{dx}{dt} = 10 \text{ m/h}$

y = dist(, MQT) @ time t

$\frac{dy}{dt} \stackrel{?}{=} -15 \text{ m/h}$

$\boxed{x = 10t}$

3 $\boxed{y = -15t+30}$

$y = -15t + b$

need $y = 0$ when $t = 2$

$0 = -15(2) + b$

$30 = b$

TIME: 4:30 pm

$t \approx 1.5$

$D' = \frac{1}{2}\left((10t)^2 + (-15t+30)^2\right)^{-1/2} \cdot \left(2(10t)\cdot 10 + 2(-15t+30)(-15)\right) = 0$

$-30(-15t+30)$

$0 = \frac{\left(2(10t)\cdot 10 + 2(-15t+30)(-15)\right)}{2\left((10t)^2 + (-15t+30)\right)^{1/2}}$ cross mult. $200t + 450t - 900 = 0$

$650t - 900 = 0$, $t = \frac{900}{650} = \frac{90}{65} = \frac{18}{13} \approx 1.5$

#19 Guide

② set $\frac{dy}{dx} = 0$ ① Find Slope $= \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

19. Find all points (x, y) where the curve $(x-1)^2 + (y-1)^2 = 1$ has a horizontal tangent.

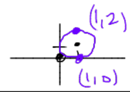
$$\frac{d}{dx}\left((x-1)^2 + (y-1)^2\right) = \frac{d}{dx}(1)$$

piece by piece w/ chain rule

$(x-h)^2 + (y-k)^2 = r^2$

r = radius

center $= (h, k)$



③ $2(x-1) + 2(y-1)\cdot\frac{dy}{dx} = 0$

④ isolate $\frac{dy}{dx} = \frac{-2(x-1)}{2(y-1)} = \frac{-(x-1)}{y-1}$

⑤ set $= 0$: $-\frac{(x-1)}{y-1} = \frac{0}{1} \Rightarrow -(x-1) = 0$

$x = 1$

⑥ sub $x = 1$ into OG

$(1-1)^2 + (y-1)^2 = 1$

$(y-1)^2 = 1$

$y - 1 = \pm 1$

$y = 1 \pm 1$

$y = 2$ or $y = 0$

Answer

⑦ $(1,0)$, $(1,2)$

20. There are two tangent lines to the curve $x^2 + xy = 1$ that have slope equal to -2. Find equations for them.

Tangent Lines

slope = derivative $\frac{dy}{dx}$ @ given x $\underline{\underline{⑤\ \text{slope}}}$ -2

① point = (x, y) $(1,0)$, $(-1,0)$

where x is (usually) given

y corresponds to this x

$y - 0 = -2(x-1)$

$y - 0 = -2(x+1)$

$y = -2x + 2$

$y = -2x - 2$

---

② $\frac{d}{dx}\left(x^2 + xy\right) = \frac{d}{dx}(1)$ product!

$2x \cdot \underbrace{\frac{dx}{dx}}_{=1} + 1 \cdot y + x \cdot \frac{dy}{dx} = 0$ isolate

slope $\frac{dy}{dx} = \frac{-2x - y}{x}$ slope formula, set $= -2$

③ $\frac{-2x - y}{x} = -2$

$\Rightarrow -2x - y = -2x$

$+2x \qquad +2x$

$y = 0$

this $\Leftrightarrow$ to two x's

④ sub into O.G.

$x^2 + x \cdot 0 = 1$

$x^2 = 1$

$x = \pm 1$

21. Find the equation of the tangent line to the graph of $y = (x^2 + 1)\sin x$ at $x = 0$. Use the linearization to approximate ~~$(1.05)^{10}$~~.

product

$$(1.05^2+1)\cdot \sin(1.05)$$

Tan Line;

slope = deriv = $\frac{dy}{dx} = 2x \cdot \sin X + (x^2+1)\cos X$

$$\left.\frac{dy}{dx}\right|_{x=0} = 2\cdot 0 \cdot \sin 0 + (0^2+1)\cdot \cos 0 = 1$$

point,

$x=0$

$y = (0^2+1)\underbrace{\sin 0}_{=0} = 0$

$(0,0)$

$$y = x$$

2

$$L(x) = x$$

$$(1.05^2+1)\cdot \sin(1.05) \approx 1.05$$

22. Suppose the volume of a spherical balloon increases at a rate of $24 \frac{cm^3}{sec}$. Find the rate that its diameter is increasing when the diameter is $3cm$.

① $V = \frac{4}{3}\pi r^3$, given: $\frac{dV}{dt} = 24 \frac{cm^3}{sec}$

vol. of sphere

② To get diam, find radius & mult by 2

$\delta = \text{diam}$
$\delta = 2r$ $\Big\}$ $\frac{d\delta}{dt} = 2\frac{dr}{dt}$

③ get $\frac{dr}{dt}$ from ① by hitting w/ $\frac{d}{dt}$

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

④ Plug in given ($\delta = 3 \Rightarrow 2r = 3$, $r = 1.5$)

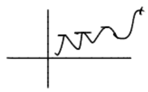
$24 = 4\pi r^2 \cdot \frac{dr}{dt}$

$= 4\pi (1.5)^2 \cdot \frac{dr}{dt}$

⑤ Isolate $\frac{dr}{dt} = \frac{24}{4\pi(1.5)^2}$

$\frac{d\delta}{dt} = 2 \cdot \frac{dr}{dt} = \frac{12}{\pi} \cdot \frac{4}{9} = \frac{48}{9\pi} \frac{cm}{s}$

23. Find the absolute maximum and absolute minimum of the function on the indicated interval.



$$f(x) = \frac{x^4}{4} - 2x^2 + 1, \quad [-3, 1] \quad \text{endpoints}$$

Plan: ① Take $f'(x)$, set $= 0$, solve : get critical pts

② compare $f(x)$ @ critical pts & endpoints

① $f'(x) = \frac{4x^3}{4} - 4x = x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0$, $x^2 - 4 = 0$

$x = \pm 2$

C.P.'s

$+2$ discard (outside interval)

-2

Compare

x	-3	-2	0	1
$f(x)$	3.25	-3	1	$-3/4$

Absolute Max:
3.25 @ x = -3

Absolute Min:
-3 @ x = -2

$\frac{(-3)^4}{4} - 2(-3)^2 + 1 = 3.25$ | $\frac{(-2)^4}{4} - 2(-2)^2 + 1$ " -3 | $\frac{0^4}{4} - 2(0)^2 + 1$ | $\frac{1}{4} - 2(1)^2 + 1$