MA 161

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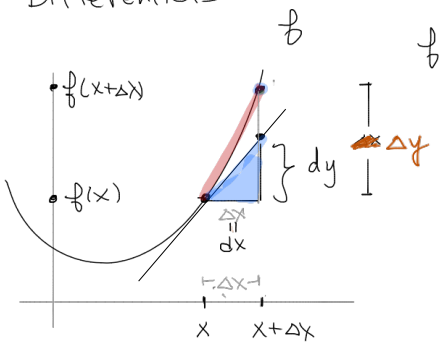
Initial Anti-Derivative Chart

Function	Anti-Derivative
x^m	$\frac{x^{m+1}}{m+1} + C$
e^x	$e^x + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$
$\sin(x)$	$-\cos(x) + C$
$\cos(x)$	$\sin(x) + C$
$\sec^2(x)$	$\tan(x) + C$
$\frac{1}{1+x^2}$	$\tan^{-1}(x) + C$
$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\sec^{-1}(x) + C$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1}(x) + C$
$\csc^2(x)$	$-\cot(x) + C$
$\sec(x)\tan(x)$	$\sec(x) + C$

(m = constant)

kick the exponent up by one and divide by it

Differentials



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{\Delta x \to 0} \underbrace{\frac{f(x+\Delta x) - f(x)}{\Delta x}}_{\text{red slope}}$$

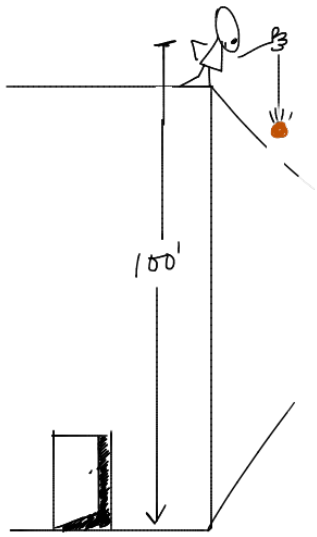
$\uparrow$ $h = \Delta x$ small change in x

$\frac{\Delta x}{\Delta y}$ = slope of secant line

$\frac{dy}{dx}$ = slope of tangent line = $f'(x)$

The point of this discussion is this: both dx and dy are "real", "physical" quantities (heights and widths) that can be multiplied and divided.

Initial Value Problem



Suppose you drop a tomato from 100 feet above the ground.

How long does it take to hit the ground?

How fast is it traveling upon impact?

} Need formula for height.

Let $f(t)$ = height of tomato @ time,

$\boxed{f(0) = 100 \text{ ft.}}$

Drop => Release

=> only Force acting is gravity, which acts as accelleration by

$\boxed{-32 \text{ ft}/s^2 = f''(t)}$

① $f'(t) = \int f''(t)\,dt = \int -32\,dt = -32t + C$

② I.V. $s(0) = 100$ (not ready to use yet)

★ $f'(0) = 0$ (we dropped it, instead of thrown it)

i.e., if we'd thrown down @ 10 ft/s,

$f'(0) = -10$

if we throw it up @ 20 ft/s,

$f'(0) = 20$

$\underset{\overset{\shortparallel}{0}}{f'(0)} = -32(0) + C \Rightarrow C = 0$

③ velocity Update: $f'(t) = -32t$

④ $f(t) = \int f'(t)\,dt = \int -32t\,dt = \frac{-32t^2}{2} + C = -16t^2 + C$

⑤ I.V. $f(0) = 100 = -16(0)^2 + C$, $C = 100$

⑥ $\boxed{f(x) = -16t^2 + 100}$ position

⑦ How long till it hits ground? Set f(x) = 0, solve. $-16t^2 + 100 = 0 \Big/ t = \frac{10}{4} = 2.5$ seconds

⑧ It takes 2.5 seconds till impact. Sub 2.5 into velocity function, you get how fast it's going at 2.5 seconds. $f'(2.5) = -32(2.5) = -80$ ft/sec

u-sub example

$$\int \underbrace{(3x+1)}_{\substack{\text{base} \\ \parallel \\ 3x+1}}{}^5 \underbrace{dx}_{\text{diff. of } x} \overset{\text{doesn't look like chart ...}}{=}$$

$$\int x^m \underbrace{dx}_{\text{differential of base}} = \frac{x^{m+1}}{m+1} + C$$

① set $\boxed{u = 3x+1}$

$\frac{du}{dx} = 3 = \frac{3}{1}$

② Isolate dx

$1 \cdot du = 3 \cdot dx$

$\boxed{\frac{1}{3} du = dx}$

③ Recopy original w/ u and dx subbed in!

$$\int (u)^5 \frac{1}{3} du = \int u^5 \cdot \frac{1}{3} du = \frac{1}{3} \int u^5 du = \frac{1}{3} \cdot \frac{u^6}{6} + C = \frac{u^6}{18} + C$$

④ get x back

$$F(x) = \frac{(3x+1)^6}{18} + C$$

Check:

$$\frac{dF}{dx} = \frac{\cancel{6}(3x+1)^5}{18} \cdot 3 = (3x+1)^5$$

Slightly more complicated u-sub

$$\int x(x^2+3)^5\,dx$$

think $\int u^m\,du$

① $\boxed{u = x^2+3}$

$\frac{du}{dx} = 2x$

② Isolate dx

$du = 2x\,dx$

$\boxed{\frac{1}{2x}\cdot du = dx}$

check

$\frac{d}{dx}(\text{ans})$

$\frac{1}{12}\cdot 6(x^2+3)^5\cdot 2x$

$= (x^2+3)^5\cdot x$

sub

$$\int x\cdot(u)^5\,\frac{1}{2x}\cdot du = \frac{1}{2}\int u^5\,du = \frac{1}{6}\cdot\frac{1}{2}u^6 = \frac{u^6}{12}+C$$

$$= \frac{1}{12}(x^2+3)^6 + C$$