MA161 - Exam 2 - ~~Guide~~ **Name:** _____

February 29, 2024

Show your work!

Find $f'(x)$.

1. $f(x) = \ln 4 + e^4$

$f'(x) = 0$

2. $f(x) = -\pi x$

$-\pi$

3. $f(x) = 4e^{-x} + \tan x + 16 \ln x$

$-4e^{-x} + \sec^2(x) + \frac{16}{x}$

power e^u

4. $f(x) = (4x^7 - 5e^{4x} + \cos x)^8$

$8(4x^7 - 5e^{4x} + \cos x)^7 (28x^6 - 20e^{4x} - \sin(x))$

5. $f(x) = (\ln x)^6$

$\frac{6(\ln x)^5}{x}$

6. $f(x) = \frac{x + 3x^2 + 4\sqrt{x}}{\sqrt{x}}$

Hint: Algebra first.

Idea: $\frac{4+10}{2} = \frac{4}{2} + \frac{10}{2} = 7$

$\frac{3x^2}{x^{1/2}} = 3x^2 \cdot x^{-\frac{1}{2}}$

$x^{\frac{1}{2}} + 3x^{\frac{3}{2}} + 4 \rightarrow \boxed{f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{9}{2}x^{\frac{1}{2}}}$

7. $f(x) = x^6 \sec x$

product

$6x^5 \cdot \sec(x) + x^6 \sec(x)\tan(x)$

8. $f(x) = e^{\cos^2 x}$ $= e^{\cos^2(x)} = e^{(\cos(x))^2}$

think: $e^u \to e^u \cdot \frac{du}{dx}$

set $\boxed{u = (\cos(x))^2}$

$w^2 \to 2w \cdot \frac{dw}{dx}$

$$\boxed{\frac{du}{dx} = 2\cos(x)(-\sin(x))}$$

$$-2e^{\cos^2(x)}(\cos(x)\sin(x))$$

9. $f(x) = \ln(\cos^2 x)$

think $\ln(u) \longrightarrow \frac{1}{u} \cdot \frac{du}{dx}$

$u = \cos^2 x$

$\frac{du}{dx} =$

$$\frac{-1}{\cos^2(x)} 2\cos(x)\sin(x) = \frac{-2\sin(x)}{\cos(x)}$$

$$= \boxed{-2\tan(x)}$$

10. $f(x) = x^5 \sin^{-1} x$

$$5x^4 \sin^{-1} x + x^5\left(\frac{d \sin^{-1} x}{dx}\right)$$

$$5x^4 \sin^{-1} x + x^5\left(\frac{1}{\sqrt{1-x^2}}\right)$$

11. $f(x) = \sin^{-1}(x^4)$

$\sin^{-1}(u) \longrightarrow \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$

$u = x^4$

sub

$\frac{du}{dx} = 4x^3$

$$\frac{4x^3}{\sqrt{1-x^8}}$$

At a time t seconds after it is thrown up in the air, a tomato is at a height (in meters) of

$$f(t) = -4.9t^2 + 55t + 1[redacted] \qquad m$$

(a) Find the instantaneous velocity of the tomato at time $t = 1$ (include units!)

$f'(t) = -9.8t + 55$

$f'(1) \approx 45 \ \frac{m}{s}$

(b) Is the tomato going up or coming down at time $t = 4$? (justify your answer)

Is $f'(t) > 0$ when $t = 4$

$f'(4) = -4(9.8) + 55 > 0 \implies$ up

(c) How high does the tomato go?

peak will occur when $f'(t) = 0$

$f'(t) = -9.8t + 55 = 0$

$t = \frac{55}{9.8} \approx 5.6$

plug this into height fcn:

$f(5.6) = -4.9(5.6)^2 + 55(5.6) + 1$

$= 155$ m

12. $f(x) = \sqrt{x^2 - 1}$

$= (x^2-1)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x^2-1)^{-\frac{1}{2}}(2x) = x(x^2-1)^{-\frac{1}{2}} = \frac{x}{\sqrt{x^2-1}}$

13. $f(x) = \ln(\ln x)$

think: $\ln(u) \longrightarrow \frac{1}{u} \cdot \frac{du}{dx} = \frac{1}{\ln(x)} \cdot \frac{1}{x}$

$u = \ln(x)$

$\frac{du}{dx} = \frac{1}{x}$

sub

$= \frac{1}{x \ln x}$

14. $f(x) = \dfrac{1-e^x}{1+e^x} \longrightarrow \dfrac{(1+e^x)(-e^x) - (1-e^x)(e^x)}{(1+e^x)^2}$

quotient

$$= \frac{-e^x - e^{2x} - (e^x - e^{2x})}{} = \frac{-e^x - e^{2x} - e^x + e^{2x}}{}$$

$$= \boxed{\frac{-2e^x}{(1+e^x)^2}}$$

15. $f(x) = \dfrac{1}{\sin(2x)} = \csc(2x)$

quotient

$$\frac{0 - 1\cdot\cos(2x)\cdot 2}{\sin^2(2x)} = \frac{-2\cos(2x)}{\sin^2(2x)} = \frac{-2\cos(2x)}{\sin(2x)\sin(2x)}$$

$= \dfrac{1}{\sin} = \csc$ $\boxed{\frac{d}{dx}(\csc(2x)) = -\csc(2x)\cot(2x)\cdot 2}$ $= -2\csc(2x)\cot(2x)$

16. There are two tangent lines to the curve $x^2 + xy = 1$ that have slope equal to -2. Find equations for them.

product

$\boxed{\text{Find } y'}$ Implicit Diff: $2x + 1\cdot y + x\cdot y' = 0$

$\boxed{\text{Isolate } y'}$

$y' = \dfrac{-2x - y}{x} = -2$

slope

set $= -2$

cross mult.

$-2x - y = -2x$

$\boxed{y = 0}$

(sub into original

$x^2 + x(0) = 1$

$\Rightarrow x^2 = 1$, $\boxed{x = \pm 1}$

points:

$(1, 0)$, $(-1, 0)$

Slope: $m = -2$

$y - y_1 = -2(x - 1)$

$\boxed{y = -2(x-1)}$

$\boxed{y = -2(x+1)}$

17. I tell my friends this class _____