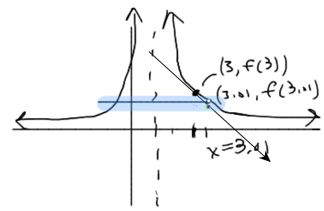
MA 161 - Week 8

1. 4.1 Linearization
2. 4.2 Critical Points & Extreme Values & 1st derivative test
3. 4.4 Concavity & 2nd derivative test
4. 4.5 L'Hopital's Rule
5. Graphing
6. Optimization

BTW
$f(3.01) = 0.247$



(b) $L(3.01)$
$=$
$-1.587(3.01) + 5.012$
$=$
0.2339

Linearization: (just the equations of the tangent line @ a point

$f(x) = \frac{1}{(x-1)^2}$ (a) Give the linearization of $f(x)$ @ $x = 3$.

$= (x-1)^{-2}$ (b) Use this to approximate $f(3.01)$.

Need: slope $= f'(x) = -2(x-1)^{-3}$ @ $x=3$: $f'(3) = -2(3-1)^{-3} = \frac{-2}{3\sqrt{2}} = -1.587$

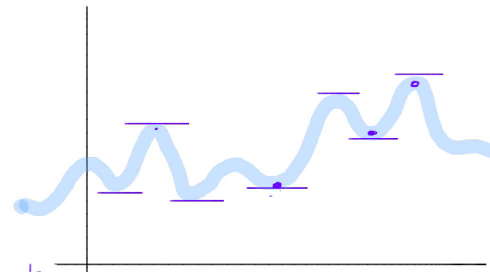
point: $x_1 = 3$
$y_1 = f(3) = \frac{1}{(3-1)^2} = \frac{1}{4}$

$\boxed{L(x) = -1.587x + 5.012}$

$y - y_1 = m(x - x_1) \rightarrow y - \frac{1}{4} = -1.587(x-3) \rightarrow y = -1.587x + 3(1.587) + .25$

Critical Points

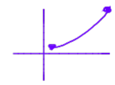
- "Candidates for where max & min occur"
- x-values (where $f'(x) = 0$ or DNE)

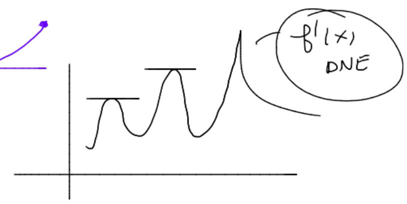


Extreme Values

- y-values (heights)

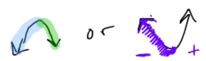
global extrema: largest (smallest) overall
- local extrema
- end point





local extrema: local min @ $x = c$ if $f'(c) = 0$ and $(-, +)$ type

$(+, -)$



$-$ $+$ 0

(order matters)

THE FIRST DERIVATIVE TEST

$f(x)$ has a local min @ $x=c$ if

- $f'(c)=0$
- $-$ $f'(a)<0$ if $a<c$
- $+$ $f'(a)>0$ if $a>0$

local max @ $x=c$ if

- $f'(c)=0$
- $+$ $f'(a)>0$ if $a<c$
- $-$ $f'(a)<0$ if $a>0$

Ex

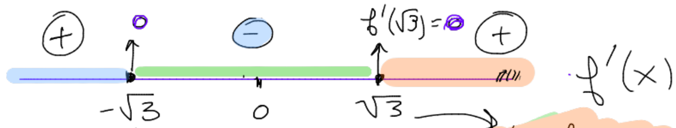
Find the absolute extrema of

$$f(x) = x^3 - 9x \quad \text{on} \quad [-1, 10]$$

① Find local extrema $\longrightarrow$ $\boxed{f'(x) = 3x^2 - 9} = 0 \Rightarrow x^2 = 3$

$x = \pm\sqrt{3}$

critical points



$-\sqrt{3} \rightarrow$ local max @ $x = -\sqrt{3}$ of $y = f(-\sqrt{3})$

$= (-\sqrt{3})^3 + 9\sqrt{3} = 10.3$

$\sqrt{3} \rightarrow$ local min @ $x = \sqrt{3}$ of $f(\sqrt{3}) = (\sqrt{3})^2 - 9(\sqrt{3})$

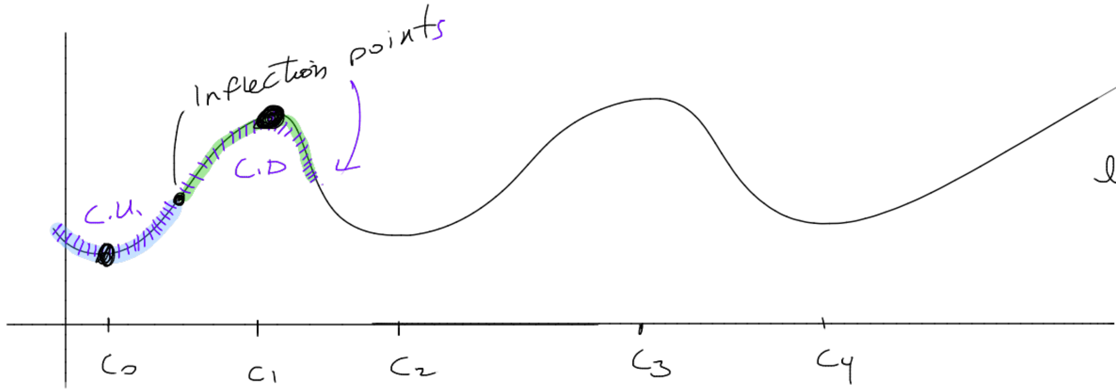
$= -10.3$ global min

② Evaluate @ endpoints

$f(-1) = -1^3 - 9(-1) = 8$, $f(10) = 10^3 - 9 \cdot 10 = 910$ absolute (global) max

③ Compare ① vs ②

2nd Derivative Test & Concavity



local min:

$f'(c)=0$

$f''(c)>0$

C.U. = concave up = $f''(x) > 0$

C.D. = concave down = $f''(x) < 0$

2nd Deriv. Test.

local Max @ $x=c$ if

$f'(c)=0$

$f''(c)<0$