MA 161 - Week 8

1. 4.1 Linearization
2. 4.2 Critical Points & Extreme Values & 1st derivative test
3. 4.4 Concavity & 2nd derivative test
4. L'Hopital's Rule
5. Graphing
6. Optimization

1. Warm-up:

equation of the tangent line @ given point

(a) Give the linearization of $\frac{1}{x} = f(x)$ @ $x = 3$.

(b) Use the linearization to approximate $\frac{1}{3.01}$.

(a) Need: slope = derivative @ $x=3$.

$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}\left(x^{-1}\right) = -x^{-2}$

@ $x=3$ $\Rightarrow$ $-\frac{1}{3^2} = -\frac{1}{9}$

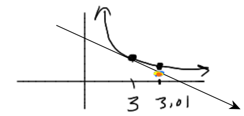
point: $x_1 = 3$

$y_1 = \frac{1}{3}$ plug into original

$\boxed{L(x) = -\frac{1}{9}x + \frac{2}{3}}$

$\Rightarrow$ $y - y_1 = m(x - x_1)$

$y - \frac{1}{3} = -\frac{1}{9}(x-3) \Rightarrow y = -\frac{1}{9}x + \frac{2}{3}$



(b) $\frac{1}{3.01} = f(x)$

$L(3.01)$

$\shortparallel$

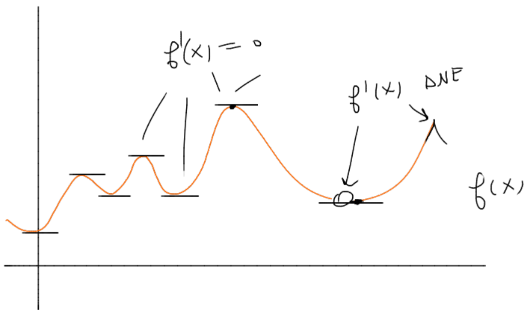
$-\frac{1}{9}(3.01) + \frac{2}{3}$

$.33\overline{2}$

vs

$\frac{1}{3} = .33\overline{3}$

CRITICAL POINTS — EXTREME VALUES — 1ST DERIV. TEST



Critical Points: (x – values) where the $\boxed{1^{st} \text{ derivative } = 0}$ or isn't defined

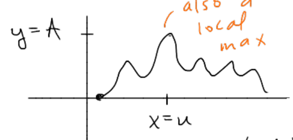
Extreme Values (y-values)

local extrema (max/min) = how high heights that correspond to critical points

local max: bigger than all nearby y-values

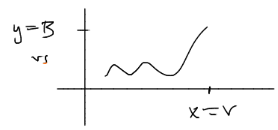
local min: smaller the all nearby y-values

global extrema (max y-value & min y-value)



global max @ (u, A)

vs



global max (v, B) not local max

Ex

$f(x) = x^4 - 16x^3$

Find all critical points, local max/min & global extrema on $[-1, 4]$.

① $f'(x) = 4x^3 - 48x^2 = 0$

$4x^2(x - 12) = 0$

$\boxed{\text{zero property}}$ $4x^2 = 0$, $x = 0$

$x - 12 = 0$, $x = 12$

$\boxed{\text{C.P.'s: } x = 0,\ x = 12}$

② Local extrema:

$f(0) = 0$

$f(12) \Rightarrow$ since $12 \notin [-1, 4]$ exclude

1st Derivative Test

$f'(-1) = 4(-1)^2(-1-12)$
$= 4(-13) = -52$

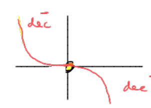
$f'(2) = 4 \cdot 2^2 (2-12) < 0$
$16(-10) = -160$

$-$ $x=0$ $-$

$f'(x)$

$f'(1) = 4 \cdot 1^2(1-12) = 4(-11) = -44$

since the critical point is of $-, -$ type $\Rightarrow$ it is not a local max or min



③ Test / Sub critical points & endpoints of region

- $f(0) = 0$
- $f(-1) = (-1)^4 - 16(-1)^3 = 17$
- $f(4) = 4^4 - 16 \cdot 4^3 = 4^3(4 - 16)$
  $= 64(-12)$
  $= -648$

④ Global Max: 17 achieved @ $x = -1$

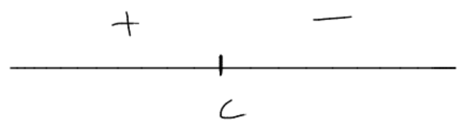
global min -648 @ $x = 4$

THE 1^{st} DERIVATIVE TEST

$f(x)$ has a

local max @ $x=c$ if

① $f'(c)=0$

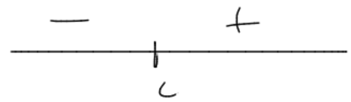
② 

$f'(x)$

type $(+,-)$

local min @ $x=c$ if

① $f'(c)=0$

② 

type $(-,+)$

Last example

$f(x) = x^3 - 8x$.

Find all local max & min.

$f'(x) = 3x^2 - 8$

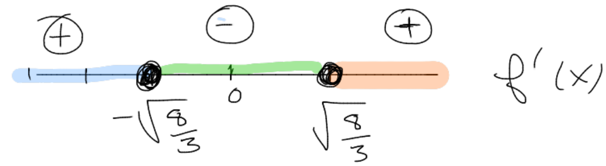
$f'(x) = 0 = 3x^2 - 8$

$8 = 3x^2$

$\frac{8}{3} = x^2$

$\pm\sqrt{\frac{9}{3}} \approx \pm\sqrt{\frac{8}{3}} = x$

critical points!



Local min @ $\underline{\sqrt{8/3}}$ of $f\left(\sqrt{\frac{8}{3}}\right) =$ ___

local max @ $\underline{-\sqrt{8/3}}$ of $f\left(-\sqrt{\frac{8}{3}}\right) =$ ___