The – wk 8

Questions ?

Study Guide:
#6

MA161 - Exam 2 - Guide **Name:** ____________________

October 14, 2024

Show your work!

Find $f'(x)$.

1. $f(x) = \ln 4 + e^4$

2. $f(x) = -\pi x$

3. $f(x) = 4e^{-x} + \tan x + 16 \ln x$

4. $f(x) = (4x^7 - 5e^{4x} + \cos x)^8$

5. $f(x) = (\ln x)^6$

6. $f(x) = \dfrac{x + 3x^2 + 4\sqrt{x}}{\sqrt{x}}$ $= x^{\frac{1}{2}} + 3x^{1.5} + 4 \rightsquigarrow$ $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + 4.5x^{\frac{1}{2}}$

$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$

7. $f(x) = x^6 \sec x$

8. $f(x) = 6^{\ln x}$

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6. $f(x) = \dfrac{x + 3x^2 + 4\sqrt{x}}{\sqrt{x}}$ Hint: Algebra first.

7. $f(x) = x^6 \sec x$

8. $f(x) = 6^{\ln x}$

9. $f(x) = e^{\cos^2 x}$

10. $f(x) = \ln(\cos^2 x)$

11. $f(x) = x^5 \sin^{-1} x$

12. $f(x) = \cos^{-1}(x^4)$

At a time t seconds after it is thrown up in the air, a tomato is at a height (in meters) of

$$f(t) = -4.9t^2 + 55t + 1m.$$

(a) Find the instantaneous velocity of the tomato at time $t = 1$ (include units!)

(b) Is the tomato going up or coming down at time $t = 4$? (justify your answer)

(c) How high does the tomato go?

13. $f(x) = \sqrt{x^2 - 1}$

14. $f(x) = \ln(\ln x)$

15. $f(x) = \dfrac{1 - e^x}{1 + e^x}$

16. $f(x) = \dfrac{1}{\sin(2x)}$

17. $f(x) = x^{7x}$

18. A boat leaves Marquette at 3:00 PM and travels due north at a speed of 10 m/h. Another boat has been heading west at 15 m/h and reaches Marquette at 5:00 PM. At what time were the boats closest together?

19. Find all points (x, y) where the curve $(x-1)^2 + (y-1)^2 = 1$ has a horizontal tangent.

20. There are two tangent lines to the curve $x^2 + xy = 1$ that have slope equal to -2. Find equations for them.

21. Find the equation of the tangent line to the graph of $y = (x^2 + 1)\sin x$ at $x = 0$. Use the linearization to approximate $(1.05)^{10}$.

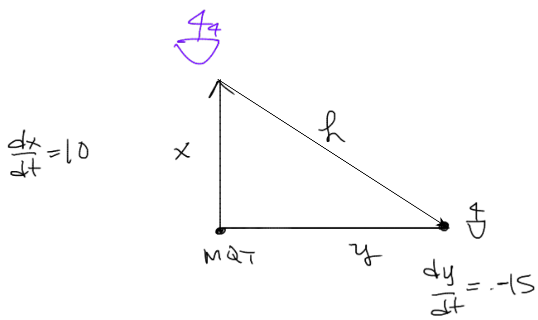
22. Suppose the volume of a spherical balloon increases at a rate of $24\frac{cm^3}{sec}$. Find the rate that its diameter is increasing when the diameter is $3cm$.

23. Find the absolute maximum and absolute minimum of the function on the indicated interval.

$$f(x) = \frac{x^4}{4} - 2x^2 + 1, \quad [-3, 1]$$

18. A boat leaves Marquette at 3:00 PM and travels due north at a speed of 10 m/h. Another boat has been heading west at 15 m/h and reaches Marquette at 5:00 PM. At what time were the boats closest together?

goal: when is h a minimum (find a local min of h)



$h^2 = x^2 + y^2$

$h = \sqrt{x^2 + y^2}$

need formulas for x, y

$h = \sqrt{(10t)^2 + (-15t+30)^2}$

$x = 10t$

$y = -15t + b$ (sub: $t = 2$ (5:00), $y = 0$)

$0 = -15(2) + b$

$30 = b$

$y = -15t + 30$

y = dist b/w MQT & boat

x = dist b/w MQT & boat, x = rate · time $= 10 \cdot t$, t = # hours since 3:00pm

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Next, Find $\frac{dh}{dt}$, set $= 0$, solve, this # is when the two boats were closest (local min)

$$\frac{dh}{dt} = \frac{1}{2}\left((10t)^2 + (-15t+30)^2\right)^{-1/2} \cdot \left(2(10t)' \cdot 10 + 2(-15t+30)'(-15)\right) = 0$$

goes down stairs

$$= \frac{200t - 30(-15t+30)}{2\left((10t)^2 + (-15t+30)^2\right)} = \frac{650t - 900}{\text{stuff}} = 0$$

($-30(-15t+30) = +450t - 900$)

$\Rightarrow 650t - 900 = 0$, $t = \frac{900}{650} = \frac{90}{65} = \frac{18}{13}$ ≈ 1.5 h

$\approx$ 4:30 pm

Review: Inc/Dec, Concavity & Points of Inflection

(section 4.4)

given: $f(x) = x^7 - x^5$

determine the intervals where $f(x)$ is:

increasing (+) $(-\infty, -\sqrt{\frac{5}{7}}) \cup (\sqrt{\frac{5}{7}}, \infty)$

decreasing $(-\sqrt{\frac{5}{7}}, 0) \cup (0, \sqrt{\frac{5}{7}})$

concave up $(-\sqrt{\frac{10}{21}}, 0) \cup (\sqrt{\frac{10}{21}}, \infty)$

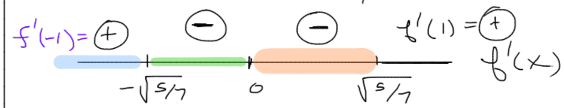
concave down $(-\infty, -\sqrt{\frac{10}{21}}) \cup (0, \sqrt{\frac{10}{21}})$

$f'(x) = 7x^6 - 5x^4 = 0$

$x^4(7x^2 - 5) = 0$

$x^4 = 0$, $7x^2 - 5 = 0$

$x = 0$ $x = \pm\sqrt{5/7}$



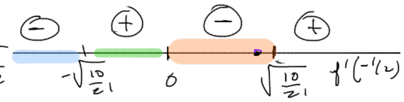
$f'(\frac{1}{2}) = (\frac{1}{2})^4(7(\frac{1}{2})^2 - 5)$

$= \frac{1}{16}(\frac{7}{4} - 5) < 0$

$f''(x) = 42x^5 - 20x^3 = 0$, $x^3(42x^2 - 20) = 0$ $f'(1/2) = \frac{1}{8}(\frac{42}{4} - 20)$

$x^3 = 0$, $42x^2 - 20 = 0$

$x = 0$, $x = \pm\sqrt{\frac{20}{42}} \approx \pm\sqrt{\frac{20}{40}} = \pm\sqrt{\frac{1}{2}}$



$f'(-1/2) = -\frac{1}{8}(-)$

L'Hopital's Rule

(limits: $\lim \frac{f(x)}{g(x)}$)

① Applies only to limits of type: $\frac{0}{0}$ or $\frac{\infty}{\infty}$

② Replace $\frac{f(x)}{g(x)}$ with $\frac{f'(x)}{g'(x)}$

Ex $\lim_{x\to\infty} \frac{15x^4 + 7}{5x^4 + 100} \overset{D.S.}{=} \frac{15\cdot\infty^4 + 7}{5\cdot\infty^4 + 100} = \frac{\infty}{\infty}$ L'H applies

" $\lim_{x\to\infty} \frac{60x^3}{20x^3} = \lim \frac{60}{20} = \lim_{x\to\infty} 3 = 3$

Ex

$\lim_{x\to 0} \frac{\cos x - 1}{x} = \frac{\cos 0 - 1}{0} = \frac{1-1}{0} = \frac{0}{0}$ L'H applies

||

$\lim_{x\to 0} \frac{-\sin x}{1} = \frac{-\sin 0}{1} = \frac{0}{1} = 0$

Ex $\lim_{x\to 0} \frac{\sin x}{x} = \frac{0}{0} \xrightarrow{L'H} \lim_{x\to 0} \frac{\cos x}{1} = \frac{\cos 0}{1} = \frac{1}{1} = 1$

Careful only $\frac{0}{0}$ or $\frac{\infty}{\infty}$

indeterminate form

Ex $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x = \left(1 + \frac{1}{\infty}\right)^\infty = 1^\infty \neq 1$ ($\frac{1}{\infty} \to 0$)

① set $y = \lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x$, goal: find y

② hit w/ ln

$\ln(y) = \ln\left(\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x\right)$

$= \lim_{x\to\infty} \left(\ln\left(1 + \frac{1}{x}\right)^x\right)$

$= \lim_{x\to\infty} x \cdot \ln\left(1 + \frac{1}{x}\right) = \infty \cdot \ln\left(1 + \frac{1}{\infty}\right)$

$= \infty \cdot \ln(1)$

$= \infty \cdot 0$ indet. form

$= \lim_{x\to\infty} \frac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}} = \frac{\ln(1)}{\frac{1}{\infty}} = \frac{0}{0}$ (L'H)

$\ln(y) = \lim_{x\to\infty} \frac{\left[\frac{-\frac{1}{x^2}}{1 + \frac{1}{x}}\right]}{-\frac{1}{x^2}}$ $\left(\cdot \frac{-x^2}{-1}\right)$ $= \lim_{x\to\infty} \frac{1}{1 + \frac{1}{x}} = \frac{1}{1+0} = 1$

$y = e^{\ln(y)} = e^1$

$y = e$