Wk 8 - Tue

Question 1 of 8

The dosage D of diphenhydramine for a dog of body mass w kg is $D = 4.7w^{2/3}$ mg. Estimate the maximum allowable error in w for a cocker spaniel of mass $w = 10$ kg if the percentage error in D must be less than 3.75%.

(Use decimal notation. Give your answer to two decimal places.)

idea: $D = 100$, $\Delta D = 5$, $\frac{5}{100} = \frac{1}{20}$ % error

$\Delta w =$ ______ kg

Key #1 $D' \approx \frac{\Delta D}{\Delta w}$

$D' = 4.7 \cdot \frac{2}{3} w^{-1/3} = \frac{\Delta D}{\Delta w}$

Key #2 $3.75\% = .0375 = \frac{\Delta D}{D}$

isolate ΔD

Plan

(1) $\underbrace{4.7 \cdot \frac{2}{3} w^{-1/3}}_{w=10} \cdot \underbrace{\Delta w}_{\text{goal}} = \Delta D$

(2) sub (1) into

$.0375 = \frac{4.7 \cdot \frac{2}{3} w^{-1/3} \cdot \Delta w}{D}$

$= \frac{4.7 \cdot \frac{2}{3} w^{-1/3} \Delta w}{4.7 w^{2/3}}$

(3) isolate Δw

$.0375 = \frac{2 \Delta w}{3 w} = \frac{2 \Delta w}{3 \cdot 10}$

#19 Guide

① Find slope $= \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$

② set $\frac{dy}{dx} = 0$

19. Find all points (x, y) where the curve $(x-1)^2 + (y-1)^2 = 1$ has a horizontal tangent.

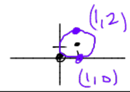
$$\frac{d}{dx}\left((x-1)^2 + (y-1)^2\right) = \frac{d}{dx}(1)$$

piece by piece w/ chain rule

$(x-h)^2 + (y-k)^2 = r^2$

$r =$ radius

center $= (h, k)$



③ $2(x-1) + 2(y-1)\cdot\frac{dy}{dx} = 0$

④ isolate $\frac{dy}{dx} = \frac{-2(x-1)}{2(y-1)} = \frac{-(x-1)}{y-1}$

⑤ set $= 0$: $\frac{-(x-1)}{y-1} = \frac{0}{1} \Rightarrow -(x-1) = 0$

$x = 1$

⑥ sub $x = 1$ into OG

$(1-1)^2 + (y-1)^2 = 1$

$(y-1)^2 = 1$

$y - 1 = \pm 1$

$y = 1 \pm 1$

$y = 2$ or $y = 0$

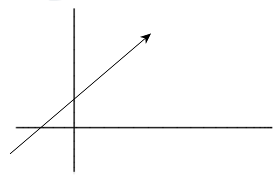
⑦ $(1,0)$, $(1,2)$

Concavity & the 2nd Derivative Test

(4-3)

More nuanced take on increasing / decreasing.

INC



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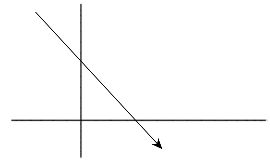
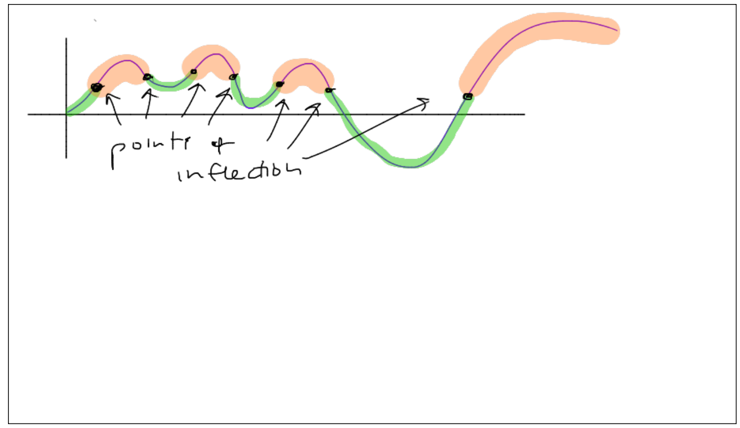
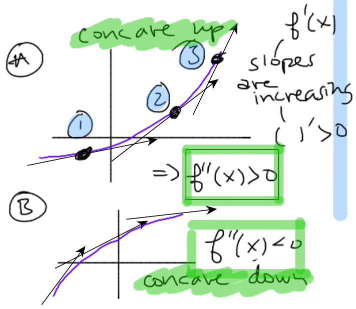
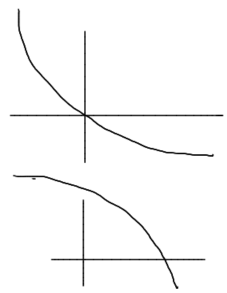








Ex Determine the regions where $f(x)$ is concave up / down & give P.O.I's

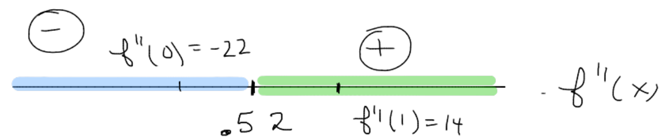
$$f(x) = 6x^3 - 11x^2 + 2$$

$$f'(x) = 18x^2 - 22x$$

$$f''(x) = 36x - 22 \overset{①}{=} 0$$

$$36x = 22$$

$$x = \frac{22}{36} = \frac{11}{18} \approx .52$$



C.U.: $(0.52, \infty)$

C.D.: $(-\infty, 0.52)$

P.O.I = $x = 0.52$

L'Hôpital's Rule: $\frac{f(x)}{g(x)}$

1. Applies only to limits of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$

2. If limit is $\frac{0}{0}$ or $\frac{\infty}{\infty}$, replace w/ $\frac{f'(x)}{g'(x)}$

Ex

$$F(x) = \frac{3x^2+1}{x+100}$$

$$\lim_{x\to+\infty} \frac{3x^2+1}{x+100} = \frac{3\cdot\infty^2+1}{\infty+100} = \frac{\infty}{\infty} \quad \text{L'H applies}$$

$$= \lim_{x\to+\infty} \frac{6x}{1} = 6\cdot\infty = \infty$$

Ex

$$\lim_{x\to\infty} \frac{3x^5+1}{6x^5+10} = \frac{\infty}{\infty}$$

L'H

$$= \lim_{x\to\infty} \frac{15x^4}{30x^4} \overset{\text{algebra}}{=} \lim_{x\to\infty} \frac{15}{30} = \frac{1}{2}$$

Ex

$$\lim_{x\to\infty} \frac{x^5+x^4}{5x^5+x^3} = \frac{\infty}{\infty}$$

L'H

$$= \lim_{x\to\infty} \frac{5x^4+4x^3}{25x^4+3x^2} = \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{20x^3+12x^2}{100x^3+6x} = \lim_{x\to\infty} \frac{60x^2+24x}{300x^2+6} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{120x+24}{600x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{120}{600} = \frac{12}{60} = \frac{1}{5}$$

<u>Careful!</u> $\frac{1}{\infty} = 0$

indeterminate form

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \left(1+\frac{1}{\infty}\right)^\infty = 1^\infty$$

① set $y = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

② hit w/ log $\ln(y) = \lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^x = \lim\left(x \cdot \ln\left(1+\frac{1}{x}\right)\right)$

ln(lim) = lim(ln)

$\infty \cdot 0$ indeterm form

L'H applies $\dfrac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \frac{0}{0}$

$$\ln(y) = \lim \left[\frac{\dfrac{-1/x^2}{1+\frac{1}{x}}}{-1/x^2}\right] = \lim_{x\to\infty}\frac{1}{1+\frac{1}{x}} = 1$$

so ln(y) = 1, thus y = e