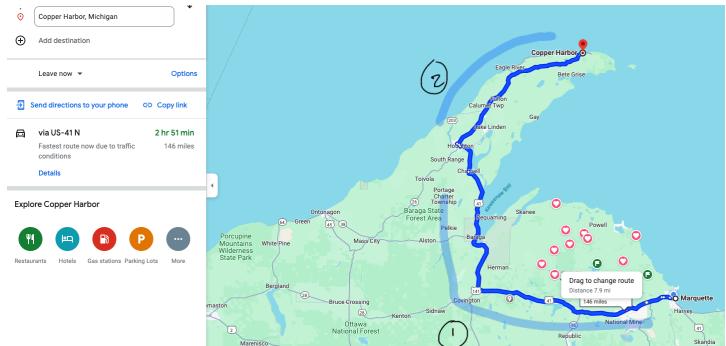


Thursday - wk 8



$$\approx 150 \text{ mi} = 50 \frac{\text{mi}}{\text{hr}} \times 3 \text{ hr}$$

↓
rate of change

$$= \frac{ds}{dt} \cdot \text{time}$$

$$\begin{aligned} \text{Total Distance} \\ (1) + (2) &= 150 \end{aligned}$$

$$\begin{array}{rcl} 100 & + & 50 \\ " & & " \\ " & & " \end{array}$$

$$60 \frac{\text{mi}}{\text{h}} \times 1.8 \text{ h} + 40 \frac{\text{mi}}{\text{h}} \times 1.1 \text{ h}$$

continue to break down

distance = $\int \text{instantaneous velocity} \cdot dt =$
 Typically, you know $\frac{\text{velocity}}{\text{rate of change}}$ at all time, you want to know
 distance traveled. So ... given

distance = $\int \text{velocity} \cdot dt$ eg if $v(t) = t^2 + 1$ is the velocity of a particle @ time t , then its

displacement (distance traveled) @ time t is

$$D(t) = \int (t^2 + 1) dt = \int t^2 dt + \int 1 dt = \frac{t^3}{3} + t + C$$

to get precise formula for displacement, we need some initial value
 know $D(0)$, or $D(1)$, or $D(2)$, eg $D(0) = 5$ given

$$D(0) = \frac{0^3}{3} + 0 + C = 5$$

$C = 5$

$$D(t) = \frac{t^3}{3} + t + 5$$

Ex 2 Let $f'(x) = x^2 + x^3$, $f(1) = 3$, find $f(x)$

Idea:

$$(1) f(x) = \int f'(x) dx = \int x^2 + x^3 dx = \frac{x^3}{3} + \frac{x^4}{4} + C$$

|
anti-derivative of the derivative

$$(2) f(1) = 3 \quad \left. \begin{array}{l} \text{if sub } x=1 \text{ into (1)} \\ \frac{1^3}{3} + \frac{1^4}{4} + C = 3 \end{array} \right\} C = 3 - \frac{1}{3} - \frac{1}{4} = 3 - \frac{7}{12} = 2\frac{5}{12} \approx 2.9$$

$$(3) f(x) = \frac{x^3}{3} + \frac{x^4}{4} + 2\frac{5}{12}$$

Ex. Let $f''(x) = x+1$, $f'(1)=2$, $f(3)=4$.

Find $f(x)$.

$$\textcircled{1} \quad f'(x) = \int f''(x) dx = \int x+1 dx$$

$$= \frac{x^2}{2} + x + C$$

$$= \frac{x^2}{2} + x + \frac{1}{2}$$

\textcircled{2} use I.V.'s initial value:

$$\begin{aligned} f'(1) &= 2 \\ f'' &= \frac{1}{2} \\ \frac{1}{2} + 1 + C &= 2 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} C = \frac{1}{2}$$

$$\textcircled{3} \quad f(x) = \int f'(x) dx = \int \frac{x^2}{2} + x + \frac{1}{2} dx = \frac{x^3}{6} + \frac{x^2}{2} + \frac{1}{2}x + D$$

$$\textcircled{4} \quad \text{I.V. } f(3)=4 \Rightarrow \frac{3^3}{6} + \frac{3^2}{2} + \frac{1}{2}(3) + D = 4 \Rightarrow \frac{9}{2} + \frac{9}{2} + \frac{3}{2} + D = 4 \Rightarrow 15 + D = 4 \Rightarrow D = -11$$

$$\textcircled{5} \quad f(x) = \frac{x^3}{6} + \frac{x^2}{2} + \frac{x}{2} - 11$$

