MA 161 wk 8 wed

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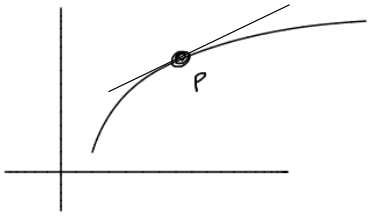
- 1. Wednesday
  - a. anti-derivatives
- 2. Thursday
  - a. anti-derivatives
- 3. Friday
  - a. video lecture online

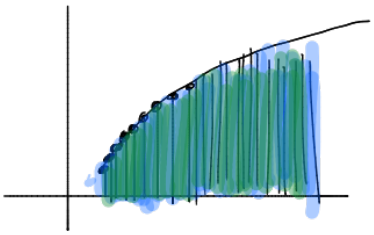
Calculus

Differentiation



compute rates of change

Integration*



compute accumulation

(need "anti-derivative"

Anti-Derivative

$\int f(x)\,dx = F(x)$ means $\frac{d}{dx}\left(F(x)\right) = f(x)$

(the x and dx are the variables that match)

Ex $\int 3x^2\,dx =$ all ~~some~~ function whose derivative with respect to x is 3x^2

$= x^3 + C$ (C is some arbitrary constant)

Ex $\int e^x\,dx = e^x + C$

Ex $\int \cos x\,dx = \sin x + C$

Ex $\int \sin(x)\,dx = -\cos x + C$

Ex $\int \frac{1}{x}\,dx = \ln|x| + C$

Ex $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ b/c $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = \frac{1}{n+1}\cdot(n+1)x^{\overbrace{n+1-1}^{=n}} = x^n$

$\frac{d}{dx}\left(\frac{x^5}{4}\right) = \frac{1}{4}\cdot 5x^4 = \frac{5}{4}x^4$

u - substitution

$$\int u^n\,du = \frac{u^{n+1}}{n+1} + C$$

- $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
- $\int c \cdot f(x)\,dx = c\int f(x)\,dx$

EX $\int (3x+1)^2\,dx \overset{\text{algebra 1st}}{=\!=} \int 9x^2 + 6x + 1\,dx$

$= \int u^2 \tfrac{1}{3}\,du = \int 9x^2\,dx + \int 6x\,dx + \int 1\,dx$

$= \tfrac{1}{3}\int u^2\,du = 9\int x^2\,dx + 6\int x^1\,dx + \int 1\,dx$

$\tfrac{1}{3}\cdot\frac{u^3}{3} = \frac{u^3}{9} + C = 9\frac{x^3}{3} + \frac{6x^2}{2} + x + C$

get x back $= \frac{(3x+1)^3}{9} + C$

$\frac{d}{dx}(5x^2 \pm 7x) = \frac{d}{dx}(5x^2) \pm \frac{d}{dx}(7x)$

$= 10x + 7$

$\frac{d}{dx}(c\,f(x)) = c \cdot \frac{d}{dx} f(x)$

Idea: transform the given problem into an equivalent, but simpler one.

dx = differential of x

du = differential of u

"small change in u"

step 1 set $u = 3x+1$

step 2 $\frac{d}{dx}$ to both: $\frac{d}{dx}(u) = \frac{d}{dx}(3x+1)$ (derivative w.r.t. x)

step 3 $\frac{du}{dx} = 3 = \frac{3}{1}$

step 4 $du = 3\,dx$

cross mult

step 5 solve for dx

$dx = \frac{1}{3}\,du$