12.5

At a time t seconds after it is thrown up in the air, a tomato is at a height (in meters) of

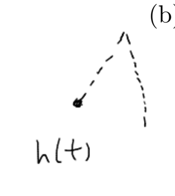
$$f(t) = -4.9t^2 + 55t + 1m.$$

(a) Find the $\underbrace{\text{instantaneous velocity}}$ of the tomato at time $t = 1$ (include units!)

$f'(t) = -9.8t + 55$

$f'(1) = -9.8(1) + 55 = 45.2 \frac{m}{s}$

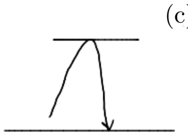
(b) Is the tomato going up or coming down at time $t = 4$? (justify your answer)



sign $(f'(4))$ $\Rightarrow$ up since $f'(4) > 0$

$f'(4) = -9.8(4) + 55 \approx -44 + 55 > 0$

(c) How high does the tomato go?



global max

set = 0

$f'(t) = -9.8t + 55 = 0$ $\quad t = \frac{55}{9.8} \approx 5.62$ sec

$f(5.62) = -4.9(5.62)^2 + 55(5.62) + 1 = 155$ meters

13. $f(x) = \sqrt{x^2 - 1}$

14. $f(x) = \ln(\ln x)$

15. $f(x) = \dfrac{1-e^x}{1+e^x}$

quotient

$$\frac{(1+e^x)(-e^x) - (1-e^x)(e^x)}{(1+e^x)^2}$$

$$= \frac{-e^x - e^{2x} - e^x + e^{2x}}{(1+e^x)^2} = \frac{-2e^x}{(1+e^x)^2}$$

16. $f(x) = \dfrac{1}{\sin(2x)}$

quotient

or

$\frac{1}{\sin} = \csc$

$$\longrightarrow \frac{-1\cdot\cos(2x)\cdot 2}{\sin^2(2x)} = -\frac{2\cos(2x)}{\sin^2(2x)} = -2\cot(2x)\cdot\csc(2x)$$

$$\csc(2x) \longrightarrow \underline{-2\csc(2x)\cot(2x)}$$

17. $f(x) = x^{7x}$

18. A boat leaves Marquette at 3:00 PM and travels due north at a speed of 10 m/h. Another boat has been heading west at 15 m/h and reaches Marquette at 5:00 PM. At what time were the boats closest together?

19. Find all points (x, y) where the curve $(x-1)^2 + (y-1)^2 = 1$ has a horizontal tangent.

20. There are two tangent lines to the curve $x^2 + xy = 1$ that have slope equal to -2. Find equations for them.

21. Find the equation of the tangent line to the graph of $y = (x^2+1)\sin x$ at $x = 0$. Use the linearization to approximate ~~[illegible]~~. $(0.5^2+1)\cdot \sin(0.5)$

similar to #20

22. Suppose the volume of a spherical balloon increases at a rate of $24\frac{cm^3}{sec}$. Find the rate that its diameter is increasing when the diameter is $3cm$.

23. Find the absolute maximum and absolute minimum of the function on the indicated interval.

$$f(x) = \frac{x^4}{4} - 2x^2 + 1, \quad [-3, 1]$$

23. Find the absolute maximum and absolute minimum of the function on the indicated interval.

$$f(x) = \frac{x^4}{4} - 2x^2 + 1, \ [-3, 1]$$

Plan: (1) take $f'(x)$, set $= 0$, solve, get Critical Points (C.P.'s)

(2) compare OG $f(x)$ on C.P.'s and endpoints

$$f'(x) = \frac{4x^3}{4} - 4x = x^3 - 4x = 0$$

$$x(x^2-4) = 0$$

$$x = 0 \qquad x^2 - 4 = 0$$

$$x = 0 \qquad x = \pm 2$$

$+2$ is outside $[-3, 1]$ exclude!

C.P.'s: $\{0, -2\}$

$$\boxed{\begin{array}{l} \text{Abs Max is } 3.25 \ @ \ x = -3 \\ \text{Abs Min is } -3 \ @ \ x = -2 \end{array}}$$

x	-3	-2	0	1
$f(x)$	3.25	-3	1	$-.75$

$f(1) = \frac{1^4}{4} - 2 + 1 = -.75$, $f(0) = 1$

$f(-3) = \frac{-3^4}{4} - 2(-3)^2 + 1 =$ $\qquad f(-2) = -3$

22. Suppose the volume of a spherical balloon increases at a rate of $24\frac{cm^3}{sec}$. Find the rate that its diameter is increasing when the diameter is $3cm$.

0 Need: $V = \frac{4}{3}\pi r^3$

vol. of sphere with radius $= r$

1 goal: $\left.\frac{dD}{dt}\right|_{D=3}$ where D = diameter

2 relate D to r,

$D = 2r \Rightarrow \frac{dD}{dt} = 2\frac{dr}{dt}$

rephrase goal:

Find $\left.\frac{dr}{dt}\right|_{D=3}$ & use that to give $\frac{dD}{dt}$

(multiply by 2)

3 Get $\frac{dr}{dt}$.

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$

$24 = \frac{dV}{dt} = 3 \cdot \frac{4}{3}\pi r^2 \cdot \frac{dr}{dt}$

4 $D = 3 = 2r$, $r = 1.5$ sub into 3

$24 = 4\pi(1.5)^2 \cdot \frac{dr}{dt}$

$$\boxed{\frac{24}{9\pi} = \frac{4 \cdot 6}{9\pi} = \frac{6}{\pi \cdot 9/4} = \frac{dr}{dt}}$$

5 $\frac{dD}{dt} = 2\frac{dr}{dt} = \frac{48}{9\pi}\ \frac{cm}{sec}$

$\frac{48}{27} \approx 1.8\ \frac{cm}{s}$

product

21. Find the equation of the tangent line to the graph of $y = (x^2 + 1)\sin x$ at $x = 0$. Use the linearization to approximate ~~f(0.5)~~. $(0.5^2 + 1) \cdot \sin(0.5)$

(just 0.5 plugged into given

similar to #20

slope: $y' = 2x \cdot \sin x + (x^2+1) \cdot \cos x$

$y'\big|_{x=0} = 2 \cdot 0 \cdot \underbrace{\sin 0}_{=0} + (0^2+1) \cdot \underbrace{\cos 0}_{=1} = 1$

point: given $x = 0$ sub into OG:

$y = (0^2+1) \cdot \overset{0}{\sin 0} = 0$

" $(0,0)$

Line: $y - 0 = 1(x - 0)$

$y = x$

approximates given function near $x = 0$

$L(x) = x$

Approx: $(0.5^2 + 1) \cdot \sin(0.5) \approx 0.5$

20. There are two tangent lines to the curve $x^2 + xy = 1$ that have slope equal to -2. Find equations for them.

Tangent Lines:
Needs point: (x, y)
slope = derivative $\frac{dy}{dx}$ @ given data (usually x)

① hit eqn w/ $\frac{d}{dx}$

$$\frac{d}{dx}(x^2 + xy) = \frac{d}{dx}(1)$$

product

$$2x \cdot \underbrace{\frac{dx}{dx}}_{=1} + 1 \cdot y + x \cdot \frac{dy}{dx} = 0$$

② isolate $\frac{dy}{dx}$ sub in given:

$$\frac{dy}{dx} = \underbrace{\frac{-2x - y}{x}}_{\text{formula for slope}} = -2$$

③ solve for y:

$\hookrightarrow$ $-2x - y = -2x$

$\hookrightarrow^{+2x}$ $-y = 0 \Rightarrow y = 0$

④ sub $y = 0$ into OG:

$$\left.\begin{array}{l} x^2 + x \cdot 0 = 1 \\ x^2 = 1 \end{array}\right\} x = \pm 1$$

points
(1, 0)
(-1, 0)

⑤ $y - 0 = -2(x - 1)$

$y = -2x + 2$

$y - 0 = -2(x - (-1))$

$y = -2x - 2$