15. $f(x) = \dfrac{1 - e^x}{1 + e^x}$

16. $f(x) = \dfrac{1}{\sin(2x)}$

17. $f(x) = x^{7x}$

hit w/ ln algorithm

① set $y = x^{7x}$, want y'

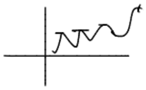
② $\ln(y) = \ln x^{7x} = 7x \cdot \ln x$ (product)

③ $\frac{d}{dx}$ $\quad \frac{1}{y} \cdot y' = 7 \cdot \ln x + \underbrace{7x\left(\frac{1}{x}\right)}_{7}$

④ $y' = (7 \cdot \ln x + 7)\, y$

⑤ $\boxed{(7\ln x + 7)\, x^{7x}}$

23. Find the absolute maximum and absolute minimum of the function on the indicated interval.



$$f(x) = \frac{x^4}{4} - 2x^2 + 1, \quad [-3, 1]$$

endpoints

Plan: ① Take $f'(x)$, set $= 0$, solve : get critical pts
② compare $f(x)$ @ critical pts & endpoints

① $f'(x) = \frac{4x^3}{4} - 4x = x^3 - 4x = 0$

$x(x^2 - 4) = 0$

$x = 0$, $x^2 - 4 = 0$

$x = \pm 2$ $+2$ discard (outside interval) -2

C.P.'s

Compare

x	-3	-2	0	1
$f(x)$	y_1	y_1	1	$-3/4$

$\frac{(-3)^4}{4} - 2(-3)^2 + 1 = y_1$ | $\frac{(-2)^4}{4} - 2(-2)^2 + 1$ " y_2 | $\frac{0^4}{4} - 2(0)^2 + 1$ | $\frac{1}{4} - 2(1)^2 + 1$

22. Suppose the volume of a spherical balloon increases at a rate of $24\frac{cm^3}{sec}$. Find the rate that its diameter is increasing when the diameter is $3cm$.

① $V = \frac{4}{3}\pi r^3$, given: $\frac{dV}{dt} = 24 \frac{cm^3}{sec}$

vol. of sphere

② To get diam, find radius & mult by 2

$\delta = \text{diam}$, $\delta = 2r$ } $\frac{d\delta}{dt} = 2\frac{dr}{dt}$

③ get $\frac{dr}{dt}$ from ① by hitting w/ $\frac{d}{dt}$

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

④ Plug in given ($\delta = 3 \Rightarrow 2r = 3$, $r = 1.5$

$24 = 4\pi r^2 \cdot \frac{dr}{dt}$

$= 4\pi (1.5)^2 \cdot \frac{dr}{dt}$

⑤ Isolate $\frac{dr}{dt} = \frac{24}{4\pi(1.5)^2}$

$\frac{d\delta}{dt} = 2 \cdot \frac{dr}{dt} = \frac{12}{\pi} \cdot \frac{4}{9} = \frac{48}{9\pi} \frac{cm}{s}$

21. Find the equation of the tangent line to the graph of $y = (x^2 + 1)\sin x$ at $x = 0$. Use the linearization to approximate ~~$(1.05)^{10}$~~.

product

$(1.05^2 + 1) \cdot \sin(1.05)$

Tan Line:

slope = deriv = $\frac{dy}{dx} = 2x \cdot \sin x + (x^2+1)\cos x$

$\frac{dy}{dx}\Big|_{x=0} = 2 \cdot 0 \cdot \sin 0 + (0^2+1) \cdot \cos 0 = 1$

point:

$x = 0$

$y = (0^2+1)\underbrace{\sin 0}_{=0} = 0$

$(0, 0)$

$y = x$

2

$L(x) = x$

$(1.05^2+1) \cdot \sin(1.05) \approx 1.05$

20. There are two tangent lines to the curve $x^2+xy=1$ that have slope equal to -2. Find equations for them.

Tangent Lines

slope = derivative $\frac{dy}{dx}$ @ given x = -2 (5) slope

(1) point = (x,y) where x is (usually) given, y corresponds to this x

$(1,0)$, $(-1,0)$

$y-0=-2(x-1)$

$y-0=-2(x+1)$

$y=-2x+2$

$y=-2x-2$

---

(2) $\frac{d}{dx}\left(x^2+xy\right)=\frac{d}{dx}(1)$ product!

$2x\cdot\frac{dx}{dx}^{=1}+1\cdot y+x\cdot\frac{dy}{dx}=0$ isolate

(slope) $\frac{dy}{dx}=\frac{-2x-y}{x}$ slope formula, set $=-2$

(3) $\frac{-2x-y}{x}=-2$

$\Rightarrow -2x-y=-2x$

$+2x \qquad +2x$

$y=0$

this $\Leftrightarrow$ to two x's

(4) sub into OG.

$x^2+x\cdot 0=1$

$x^2=1$

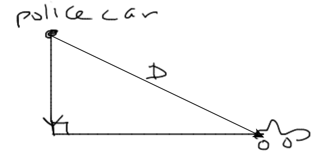
$x=\pm 1$

18. A boat leaves Marquette at 3:00 PM and travels due north at a speed of 10 m/h. Another boat has been heading west at 15 m/h and reaches Marquette at 5:00 PM. At what time were the boats closest together? $\Rightarrow$ Minimize D, local min!

Plan: Find D'
Set $D'=0$
Solve

similar to:

police car



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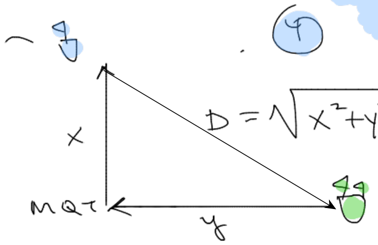

$$D=\sqrt{x^2+y^2}=\sqrt{(10t)^2+(-15t+30)^2}$$

1. let $x=$ dist(, MQT) @ $t=$ # hours since 3:00 pm
2. $\frac{dx}{dt}=10\ m/h$

$y=$ dist(, MQT) @ time t

$\frac{dy}{dt}\stackrel{?}{=}-15\ m/h$

$\boxed{x=10t}$

3. $\boxed{y=-15t+30}$

$y=-15t+b$

need $y=0$ when $t=2$

$0=-15(2)+b$

$30=b$

TIME: 4:30 pm

$t\approx 1.5$

$$D'=\frac{1}{2}\left((10t)^2+(-15t+30)^2\right)^{-1/2}\cdot\left(2(10t)\cdot 10+\underbrace{2(-15t+30)(-15)}_{-30(-15t+30)}\right)=0$$

$$0=\frac{\left(2(10t)\cdot 10+2(-15t+30)(-15)\right)}{2\left((10t)^2+(-15t+30)\right)^{1/2}}$$

cross mult. $\rightsquigarrow$ $200t+450t-900=0$

$650t-900=0$, $t=\frac{900}{650}=\frac{90}{65}=\frac{18}{13}\approx 1.5$