Week 9 - Friday

warm-up: $\int 2x - 7x^2 + \underline{9} + \frac{8}{x}\,dx$

$$\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$$

$$\int \frac{2x^2 - 7x^3 + 9x + 8}{x}\,dx = \boxed{\frac{2x^2}{2} - \frac{7x^3}{3} + 9x + 8\ln|x| + C}$$

$$9x^0 \longrightarrow \frac{9x^1}{1}$$

Evaluate the integral and check your answer by differentiating.

$$\int \left(6+3y^2\right)^2 dy = \boxed{} + C$$

expand first

$$\int 8\sqrt{x} + \frac{5}{\sqrt{x}}\,dx = \boxed{\frac{16}{3}x^{3/2} + 10x^{\frac{1}{2}}} + C.$$

$$-\frac{1}{2}+1$$

$$\int 8x^{\frac{1}{2}} + 5x^{-\frac{1}{2}}\,dx = \frac{2}{3}\cdot 8x^{\frac{3}{2}} + \frac{2}{1}\cdot 5x^{\frac{1}{2}} + C$$

when to use u-sub:

ex $\int 8\sqrt{x+1} + \frac{5}{\sqrt{x+1}}\,dx$ x doesnt match x+1

whenever the letter that follows the d in the integral DOES NOT MATCH everything being fed into some function (power, trig)

$= e^{-x} + e^{-3x}$

This is a multi-part problem. If $F(x) = e^{-x} + \frac{1}{e^{3x}}$, find $F'(x)$.

$F'(x) =$ $e^{-x}(-1) + e^{-3x}(-3)$

$= -e^{-x} - 3e^{-3x}$

$$\int \left(-\frac{1}{e^{x}} - 3e^{-3x} \right) dx =$$

$\shortparallel$

$$\int -e^{-x} - 3e^{-3x}\, dx = -\int e^{-x}\, dx - 3\int e^{-3x}\, dx$$

$u = -x$ | $\frac{du}{dx} = -1$ | $du = -dx$

$u = -3x$ | $\frac{du}{dx} = -3$ | $du = -3\,dx$

$$= \int e^{-x}(-dx) + \int e^{-3x}(-3)\,dx$$

$$= \int e^{u}\,du + \int e^{u}\,du$$

$$= e^{u} + e^{u} + C$$

$$= e^{-x} + e^{-3x} + C$$

Review of inverse trig derivatives and anti-derivatives

$f(x) = \sec^{-1}(x)$

what is $f'(x)$?

set $y = \sec^{-1}(x)$

compose $\sec(y) = \sec(\sec^{-1}(x))$

$\sec(y) = x$

implicit diff $\frac{d}{dx}(\sec(y)) = \frac{d}{dx}(x)$

$\sec(y)\tan(y) \cdot y' = 1$

P.T. $\Rightarrow$ $\frac{\sin^2(y)}{\cos^2(y)} + \frac{\cos^2(y)}{\cos^2(y)} = \frac{1}{\cos^2(y)}$

$\tan^2(y) + 1 = \sec^2(y)$

$\tan^2(y) = \sec^2(y) - 1$

$\tan(y) = \sqrt{\underbrace{\sec^2(y)}_{x^2} - 1}$

so $y' = \frac{1}{\underbrace{\sec(y)}_{x}\tan(y)} = \frac{1}{|x|\sqrt{x^2-1}}$

$$\boxed{\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1}(x) + C}$$

Ex

$$\int \frac{1}{x\sqrt{x^4-1}}\,dx$$

think: where on chart

$$\int \frac{1}{|u|\sqrt{u^2-1}}\,dx$$

set $\boxed{u = x^2}$, so $\boxed{u^2 = x^4}$

$\frac{du}{dx} = 2x$, $du = 2x\,dx$, $\boxed{\frac{1}{2x}\cdot du = dx}$

$\boxed{\frac{1}{2}\sec^{-1}(x^2)+C}$

$\parallel$

$\frac{1}{2}\sec^{-1}(u)+C$

$\parallel$

$$= \int \frac{1}{\cancel{x}\sqrt{u^2-1}}\cdot\frac{1}{2\cancel{x}}\,du = \frac{1}{2}\int \frac{1}{x^2\sqrt{u^2-1}}\,du = \frac{1}{2}\int \frac{1}{|u|\sqrt{u^2-1}}\,du$$

still mixed variables

$\underline{u} = x^2$ & x^2 is positive

so $\underline{u} = |u|$

1. $\int \frac{5}{\sqrt{1-x^2}}\,dx =$ $5\sin^{-1}(x) + C$

NO deriv. relation

2. $\int \frac{x}{\sqrt{1-x^2}}\,dx =$

essentially same

$u = 1 - x^2$

3. $\int \frac{4x^3}{\sqrt{1-x^4}}\,dx =$ $\int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}}(-1)\,du = \frac{2}{1} u^{\frac{1}{2}} + c = -2(1-x^4)^{\frac{1}{2}} + C$

don't see deg. 3 term

4. $\int \frac{4x}{\sqrt{1-x^4}}\,dx =$ $\frac{d}{dx}(1-x^4) = -4x^3$

NO derivative relationship …..

…> arctrig

$\sec^{-1}$

$\tan^{-1}$

$\sin^{-1}$