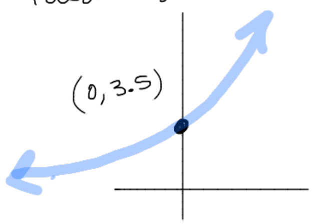
Mon. wk 9

Find the function with derivative $f'(x) = e^{2x}$ that passes through the point $P = (0, 7/2)$.

$f(x) =$ [ ]

Idea: given derivative $\boxed{f'(x)}$ and an initial value $(0, 7/2)$ ← means $\boxed{f(0) = 7/2}$



① $f(x) = \int f'(x)\,dx = \underbrace{\int e^{2x}\,dx}$

think: $\int e^u\,du$

$\boxed{u = 2x}$

$\frac{du}{dx} = 2$

↑ derivative

$du = 2\,dx$

$\boxed{\frac{1}{2}du = dx}$

② $\int e^u \frac{1}{2}du = \frac{1}{2}\int e^u\,du = \frac{1}{2}(e^u + C)$

$= \frac{1}{2}e^u + \frac{1}{2}C$

$= \frac{1}{2}e^u + D$

$\boxed{⑤\ f(x) = \frac{1}{2}e^{2x} + 3}$

③ $f(x) = \frac{1}{2}e^{2x} + D$

④ use I.V. $f(0) = \frac{1}{2}e^{2\cdot 0} + D = 3.5 \Rightarrow D = 3$

Ab.1 #1

$x^{\frac{1}{2}}$

$F(x) = \dfrac{x^8}{8} + \sqrt{x} + e^x$, find $F'(x)$.

$F'(x) =$

$\frac{1}{8} \cdot 8x^7 + \frac{1}{2}x^{-\frac{1}{2}} + e^x$

$= x^7 + .5x^{-1/2} + e^x$

$\frac{5}{2}x^{-1/2} \rightarrow$ power

$\displaystyle\int \left(-9x^7 + \frac{5}{2\sqrt{x}} - \frac{e^x}{4}\right) dx =$

$\dfrac{-9x^8}{8} + \dfrac{5x^{\frac{1}{2}}}{2\left(\frac{1}{2}\right)} - \frac{1}{4} \cdot e^x + C$

$-\frac{9}{8}x^8 + 5\sqrt{x} - \frac{1}{4}e^x + C$

AD.2 # 6

f double prime

Suppose $f''(x) = -16\sin(4x)$ and $f'(0) = -3$, and $f(0) = -3$.

$f(\pi/3) =$

Idea: (1) $f'(x) = \int f''(x)\,dx$, use (2) $f'(0) = -3$ to find c

(3) $f(x) = \int f'(x)\,dx$, use (4) $f(0) = -3$ to find c

$$f'(x) = \int -16\sin(4x)\,dx = -16\underbrace{\int \sin(4x)\,dx}_{\int \sin(u)\,du}$$

set $\boxed{u = 4x}$, $du = 4\,dx$, $\boxed{\frac{1}{4}du = dx}$

$$= -16\int \sin(u)\,\frac{1}{4}\,du$$
$$= -4\int \sin(u)\,du$$
$$= 4\cos(u) + C$$

(1) $\boxed{f'(x) = 4\cos(4x) + C}$

$\int \cos x\,dx = \sin x$

$\cos(0) = 1$

(2) $f'(0) = 4\cdot\cos(4\cdot 0) + C = -3$

$= 4 + C = -3$

$C = -7$

$\Rightarrow \boxed{f'(x) = 4\cos(4x) - 7}$

(3) $f(x) = \int f'(x)\,dx = \int 4\cos(4x) - 7\,dx = 4\int \cos(4x)\,dx - \int 7\,dx$

$\boxed{u = 4x}$, $du = 4\,dx$

$\boxed{\frac{1}{4}du = dx}$

$$= 4\int \cos(u)\cdot\frac{1}{4}\,du - \int 7\,dx$$
$$= \sin(u) - 7x + C = \sin(4x) - 7x + C$$

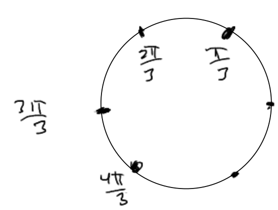
(4) $f(0) = -3$

$\sin(4x) - 7x + C = -3$

$C = -3$

Finally $\boxed{f(x) = \sin(4x) - 7x - 3}$

So $f\left(\frac{\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) - \frac{7\pi}{3} - 3$



A6.1 #3

$$\int \frac{3}{7x^5} + \frac{3x^5}{7}\,dx = \boxed{} + C.$$

$$\int \frac{3}{7x^5}\,dx + \int \frac{3x^5}{7}\,dx$$

$$\frac{3}{7}\int \frac{1}{x^5}\,dx + \frac{3}{7}\int x^5\,dx$$

$$\frac{3}{7}\int x^{-5}\,dx + \frac{3}{7}\int x^5\,dx$$

$$\frac{3x^{-4}}{-28} + \frac{3x^6}{42} + c$$

$$\int \frac{1}{x}\,dx = \ln|x| + c$$

$\int \frac{1}{x^3}\,dx$ = doesn't appear in chart

$\int x^m\,dx$ $\quad m \neq -1$