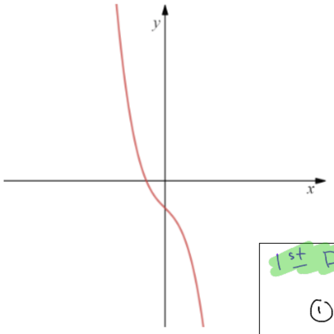
Thu - wk 9

- Finish 4-4 (and maybe 4-5 / 4-6)

Consider the graph of a function f.



Identify the correct statements about the graph.

☒ $f'(x) < 0$ for all x

☐ $f''(x) < 0$ for $x > 0$

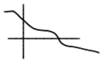
☐ $f''(x) > 0$ for $x < 0$

☐ f has one inflection point.

☐ f has one local minimum and one local maximum.

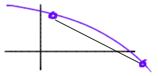
☐ $f'(x) > 0$ for all x

$f'(x) < 0 \Rightarrow$ decreasing

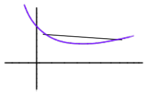


$> 0 \Rightarrow$ increasing

$f''(x) < 0 \Rightarrow$ concave down



$f''(x) > 0 \Rightarrow$ concave up



LOCAL MAX / MIN

1st Derivative Test

① $f'(c) = 0$

② $+ \wedge -$, c, $f'(x)$ $\Rightarrow$ local max

vs $-$ $+$, c, $f'(x)$ MIN

2nd derivative test

① $f'(c) = 0$

② max $f''(c) < 0$ | min $f''(c) > 0$

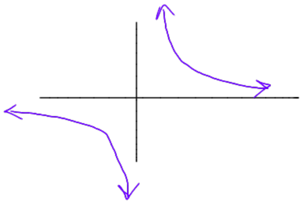
Inflection Point (concavity changes)

① $f''(c) = 0$ or DNE

② $+$ $-$, $x = c$, $f''(x)$

More on Inflection Points

Ⅰ $f(x) = \frac{1}{x}$ graph :

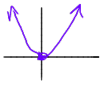


$f(x)$ has an inflection point @ $x = 0$

$f'(x) = \frac{-1}{x^2} = -x^{-2}$ ($\Rightarrow$ always decreasing)

$f''(x) = -2(-1)x^{-3} = \frac{2}{x^3}$ $\begin{cases} \text{if } x<0 \Rightarrow \frac{2}{x^3} < 0 & \text{(concave down)} \\ x > 0 \Rightarrow \frac{2}{x^3} > 0 & \text{(concave up)} \end{cases}$

Ⅱ $f(x) = x^2$, $f'(x) = 2x$, $f''(x) = 2 \Rightarrow$ always positive, f is concave up everywhere



$g(x) = x^4$, $g'(x) = 4x^3$, $g''(x) = 12x^2$

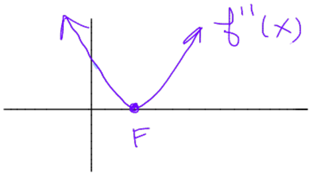
setting $12x^2 = 0 \Rightarrow x = 0$ (does this mean $x = 0$ is an inflection point ?) (NO

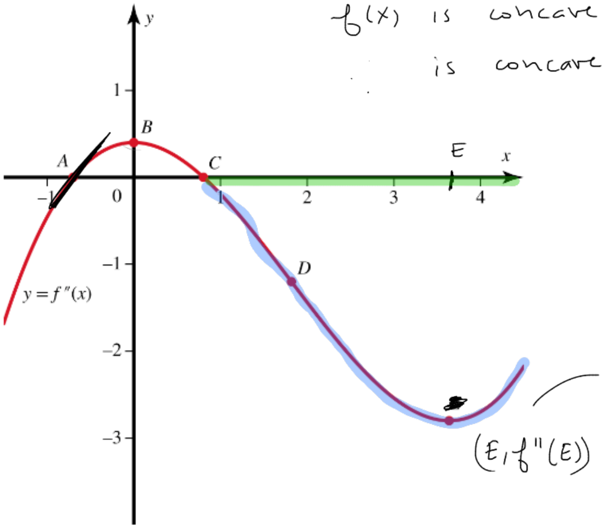
$12(-1)^2 > 0$ ⊕ , $12(1)^2 > 0$ ⊕ , $x = 0$, $f''(x)$

The graph of $y = f''(x)$ with five labeled points is shown.

Complete the sentences as to whether each labeled point is an inflection point of $y = f(x)$.

Point A ☒ an inflection point.
Point B ☐ an inflection point.
Point C ☒ an inflection point.
Point D ☐ an inflection point.
Point E ☐ an inflection point.





$f(x)$ is concave down on $(-\infty, A) \cup (C, \infty)$

is concave up on (A, C)

graph of f'' is below x-axis @ E

$\Rightarrow$

$f''(E) < 0$

4-4-3

Consider the function $f(x) = \frac{1}{x^2 + 14}$. ⇝ $f'(x) = \frac{-2x}{(x^2+14)^2}$,

↗ $f''(x) > 0$

f is concave up whenever

○ $3x^2 - 14$ is positive

○ $3x^2 - 14$ is negative

○ $3x^2 + 14$ is positive

○ $3x^2 + 14$ is negative

○ $x^2 + 14$ is negative

○ $x^2 - 14$ is negative

$$\boxed{f''(x)} = \frac{(x^2+14)^2(-2) - (-2x)(2(x^2+14)(2x))}{(x^2+14)^4}$$

$$= \frac{\cancel{(x^2+14)}\left[\overbrace{(x^2+14)(-2) + 8x^2}\right]}{(x^2+14)^4}$$

$$= \frac{-2x^2 - 28 + 8x^2}{(x^2+14)^3} = \frac{6x^2 - 28}{(x^2+14)^3} = \boxed{\frac{2(3x^2 - 14)}{(x^2+14)^3}} > 0$$

denom > 0 ⇒ mult. inequal by it

$$\Rightarrow \quad 2(3x^2 - 14) > 0$$

$$3x^2 - 14 > 0$$

9-4-10

Let $f(x) = (x^2 - 9)e^{-x} \quad (x > 0)$.

Find the critical points c that correspond to local minima.

C.P.'s

$$f'(x) = 2xe^{-x} - (x^2-9)e^{-x} = e^{-x}[2x - (x^2-9)] = e^{-x}[-x^2+2x+9] = 0$$

$$e^{-x} = 0 \quad \text{or} \quad \underbrace{-x^2 + 2x + 9}_{} = 0$$

$$\frac{1}{e^x} = 0$$

⇒ no sol's

$$-x = \ln(e^{-x}) = \ln(0)$$

$$x = -\ln(0)$$

$\boxed{\text{DNE}}$

use quad. formula to get

$$\left.\begin{array}{l} x = c_1 \\ x = c_2 \end{array}\right\} \text{one will be negative (discard)}$$

c_2 = 4.16228

Use 1st Deriv. Test

(or)

2nd Deriv. Test

$+ \wedge -$

4.1622 $f'(x)$

⇒ local max

$f''(x) =$