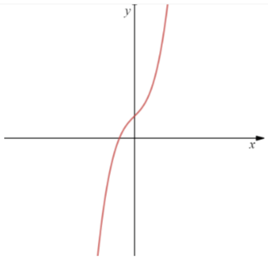
the - wk 9

Finish 4-4 / 4-5 / 4-6



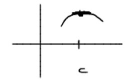
Local Max / Min

1st Derivative Test

- $f'(c) = 0$

$+ \;|\; -$ (c) $f'(x) \Rightarrow$ local max

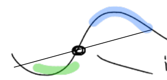
$- \;|\; +$ (c) $f'(x) \Rightarrow$ local min



2nd Derivative Test

- $f'(c) = 0$
- $f''(c) > 0 \Rightarrow$ local min
- $f''(c) < 0 \Rightarrow$ local max

mixed

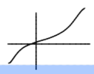


inflection point $\Leftrightarrow$ (change in concavity)

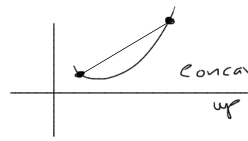
- $f''(x) = 0$
- $+ \;|\; -$ f''

Identify the correct statements about the graph.

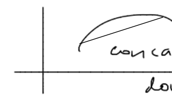
- ☐ $f'(x) > 0$ for all x $\Rightarrow$ increasing =



- ☐ $f''(x) < 0$ for $x < 0$ → concavity
- ☐ $f''(x) > 0$ for $x > 0$
- ☐ f has one inflection point. (transition point)
- ☐ f has one local minimum and one local maximum.
- ☐ $f'(x) < 0$ for all x



concave up



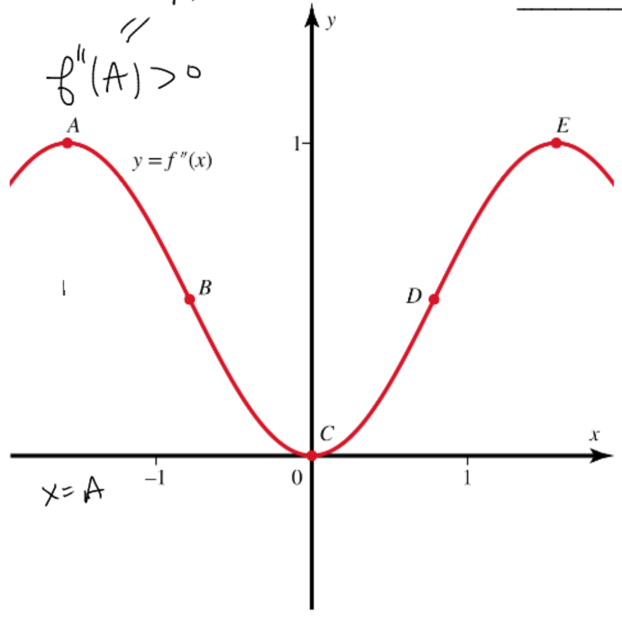
concave down

The graph of $y = f''(x)$ with five labeled points is shown.

Complete the sentences as to whether each labeled point is an inflection point of $y = f(x)$.

A is not an inflection pt

$f''(A) > 0$



$x = A$

Point A an inflection point.

Point B an inflection point.

Point C an inflection point.

Point D an inflection point.

Point E an inflection point.

Consider the function $f(x) = \dfrac{1}{x^2+14} = (x^2+14)^{-1}$, $f'(x) = -(x^2+14)^{-2} \cdot 2x = \dfrac{-2x}{(x^2+14)^2}$

$= -2x(x^2+14)^{-2}$

Solve

$f''(x) > 0$

f is concave up whenever

- ~~$3x^2 - 14$ is positive~~
- $3x^2 - 14$ is negative
- $3x^2 + 14$ is positive
- $3x^2 + 14$ is negative
- $x^2 + 14$ is negative
- $x^2 - 14$ is negative

$$f''(x) = -2(x^2+14)^{-2} - 2x\left(-2(x^2+14)^{-3}(2x)\right)$$

product rule

$$= \frac{-2}{(x^2+14)^2} + \frac{8x^2}{(x^2+14)^3}$$

common denom.

$$= \frac{\overbrace{-2(x^2+14)}^{-2x^2-28}}{(x^2+14)^3} + \frac{8x^2}{(x^2+14)^3} =$$

solve

If $3x^2 - 14 > 0$ $\Rightarrow f'' > 0$

$$\frac{3x^2 - 14}{\underbrace{(x^2+14)^3}_{\text{always } +}} > 0$$

non-linear inequality:

① set $= 0$ (or ∞) and find where DNE

② testing regions

① $6x^2 - 28 = 0$

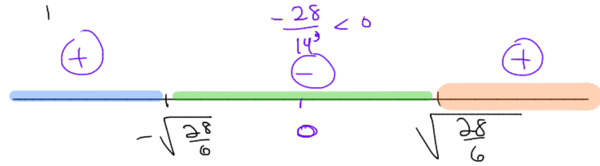
$3x^2 - 14 = 0$, $6x^2 = 28$

$x^2 = \frac{28}{6}$

$x = \pm\sqrt{\frac{28}{6}}$

and $(x^2+14)^3 = 0$ $\Rightarrow$ DNE

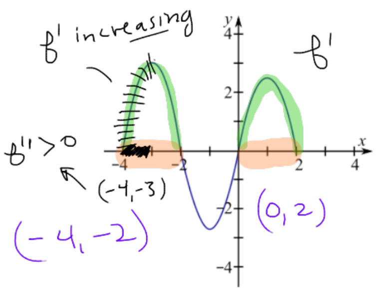
②



$f''(x)$

ANS: $\left(-\infty, -\sqrt{\frac{28}{6}}\right) \cup \left(\sqrt{\frac{28}{6}}, \infty\right)$

The figure is the graph of the derivative, f', of a function f on $[-4, 2]$.



Determine the intervals on which f is increasing.

$f' > 0$

$f'' > 0$

concave up

given: f'

want: f''

do: take derivative of f'

look for where graph of f' is increasing

#10 4-6

$$f(x) = (x^2-11)e^{-x} \qquad (x>0)$$

CRIT. PTS

$$f'(x) = 2xe^{-x} - (x^2-11)e^{-x} = e^{-x}(2x-(x^2-11))$$

$-x^2+11$

$$= e^{-x}(-x^2+2x+11) \quad ⊛$$

$$f'(x)=0 = e^{-x}(-x^2+2x+11)$$

$e^{-x} = 0$ (No sol.)

$= \frac{1}{e^x}$

$x^2-2x-11=0$

$-x^2+2x+11=0$

Quad Formula

$x_1 = c_1$

$x_2 = c_2$

x^2-4x-9

$x^2-2x-11-2x+2$

$$f''(x) ⊛= -e^{-x}(-x^2+2x+11) + e^{-x}(-2x+2) = e^{-x}\left[-(-x^2+2x+11)+(-2x+2)\right]$$

$$= e^{-x}[x^2-4x-9] = 0 \Rightarrow \begin{cases} e^{-x}=0 & \text{NONE} \\ x^2-4x-9=0 & \text{QF.} \end{cases}$$