Thurs. wk 9

Integrals Chart

$u = f(x)$

$\int u^m \, du \quad m \neq -1$	$\frac{u^{m+1}}{m+1} + C$
$\int e^u \, du$	$e^u + C$
$\int \frac{1}{u} \, du$	$\ln\lvert u\rvert + C$
$\int \sin(u) \, du$	$-\cos(u) + C$
$\int \cos(u) \, du$	$\sin(u) + C$
$\int \sec(u)\tan(u) \, du$	$\sec(u) + C$

kick it up by one
then divide by it

$\int \frac{1}{1+u^2} \, du$	$\tan^{-1}(u) + C$
$\int \frac{1}{\lvert u\rvert\sqrt{u^2-1}} \, du$	$\sec^{-1}(u) + C$
$\int \frac{1}{\sqrt{1-u^2}} \, du$	$\sin^{-1}(u) + C$

Integration technique (mining)

think: chart — $\int u^n \, du \Rightarrow$

$$\int x\sqrt{x-1}\,dx$$

set $\boxed{u = x-1}$

$\frac{du}{dx} = 1$, $\boxed{du = dx}$

sub $\int x \, u^{\frac{1}{2}} \, du$

u = f(x)

$\int u^m \, du$ $\quad m \neq -1$	$\frac{u^{m+1}}{m+1} + C$	kick it up by one then divide by it	$\cancel{\int \frac{1}{1+u^2} du}$	$\tan^{-1}(u) + C$
$\cancel{\int e^u \, du}$	$e^u + C$		$\cancel{\int \frac{1}{\lvert u\rvert\sqrt{u^2-1}} du}$	$\sec^{-1}(u) + C$
$\cancel{\int \frac{1}{u} \, du}$	$\ln\lvert u\rvert + C$		$\cancel{\int \frac{1}{\sqrt{1-u^2}} du}$	$\sin^{-1}(u) + C$
$\cancel{\int \sin(u) \, du}$	$-\cos(u) + C$			
$\int \cos(u) \, du$	$\sin(u) + C$			
$\cancel{\int \sec(u)\tan(u) \, du}$	$\sec(u) + C$			

The "x" doesn't cancel, so I "mine" the equation above "u = x-1" so isolate x

$\boxed{u+1 = x}$

$$= \int (u+1)u^{\frac{1}{2}} du = \int u^{\frac{3}{2}} + u^{\frac{1}{2}} du \overset{\text{power}}{=} \frac{u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + C$$

$$= \boxed{\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C}$$

$$\int \frac{x}{1+x^4}\,dx$$

think $\tan^{-1}(u)$

turn into . . .

$$\int \frac{1}{1+u^2}\,du$$

Check

$$\frac{d}{dx}\left(\frac{1}{2}\tan^{-1}(x^2)+C\right) = \frac{1}{2}\cdot\frac{1}{1+(x^2)^2}\cdot 2x = \frac{1}{1+x^4}\cdot x$$

$$= \frac{x}{1+x^4}$$

$\boxed{u^2 = x^4}$ square

so let $\boxed{u = x^2}$

$$\frac{du}{dx} = 2x$$

$$\boxed{du = 2x\,dx}$$

$$\boxed{\frac{1}{2x}\cdot du = dx}$$

Key: recognize 1 + x^(even #) down below

$$\xrightarrow{\text{sub}} \int \frac{\cancel{x}}{1+u^2}\,\frac{1}{2\cancel{x}}\,du = \frac{1}{2}\int\frac{1}{1+u^2}\,du = \frac{1}{2}\tan^{-1}(u)+C$$

$$= \boxed{\frac{1}{2}\tan^{-1}(x^2)+C}$$

Challenge Problems

think: chart

$u = f(x)$				
$\int u^m du$ $m \neq -1$	$\frac{u^{m+1}}{m+1} + C$	kick it up by one then divide by it	$\int \frac{1}{1+u^2} du$	$\tan^{-1}(u) + C$
$\int e^u du$	$e^u + C$		$\int \frac{1}{\lvert u \rvert \sqrt{u^2-1}} du$	$\sec^{-1}(u) + C$
$\int \frac{1}{u} du$	$\ln\lvert u \rvert + C$		$\int \frac{1}{\sqrt{1-u^2}} du$	$\sin^{-1}(u) + C$
$\int \sin(u)\, du$	$-\cos(u) + C$			
$\int \cos(u)\, du$	$\sin(u) + C$			
$\int \sec(u)\tan(u)\, du$	$\sec(u) + C$			

$$\int \tan x \, dx$$

$$=$$

☆ $$\int \frac{\sin(x)}{\cos(x)} dx$$

$$=$$

$$\int \frac{1}{\cos(x)} \cdot \sin x \, dx$$

Recognize deriv. relationship

set $\boxed{u = \cos(x)}$

$du = -\sin(x)\,dx$

$\boxed{\frac{1}{-\sin x} du = dx}$

sub $$\int \frac{\sin(x)}{u} \left(\frac{1}{-\sin(x)} \right) du = -\int \frac{1}{u} du = -\ln\lvert \cos(x) \rvert + C$$

check. $$\frac{d}{dx}\left(-\ln(\cos(x)) + c\right) = -\frac{(-\sin(x))}{\cos(x)} = \frac{\sin(x)}{\cos(x)}$$

Challenge # 2

$$\int \frac{2x+1}{x^2+1}\,dx$$

deg. 1 differences :

$u = x^2+1$

$\frac{du}{dx} = 2x$

$$\int \frac{\cancel{2x}+1}{u} \frac{1}{\cancel{2x}}\,du = \int \frac{1+\frac{1}{2x}}{u}\,du$$

mixed variables

$\Rightarrow$ stuck

try to split up!

$$\int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$\tan^{-1} x$

$$\int \frac{du}{u} + \tan^{-1} x + C = \boxed{\ln|x^2+1| + \tan^{-1}(x) + C}$$