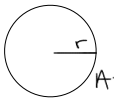
thurs wk 9

Ch 5 (5.1, 5.2) Integral Calculus

Q: how to calculate area?



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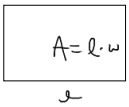

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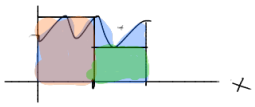
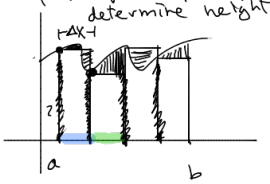


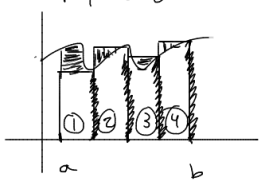
A. approximate the area w/ small rectangles, compute area of each, add
repeat, but with more (smaller) rectangles

Find area under curve $f(x)$, from a to b.

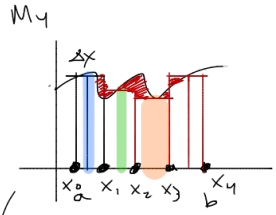
Ex L_4 (left endpoints) determine height



R_4 (right



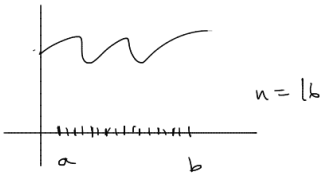
M_4



$n = 4$ (# rectangles)

using midpoint to dictate height

" $\frac{x_0 + x_1}{2}$, $\frac{x_1 + x_2}{2}$, $\frac{x_1 + x_3}{2}$



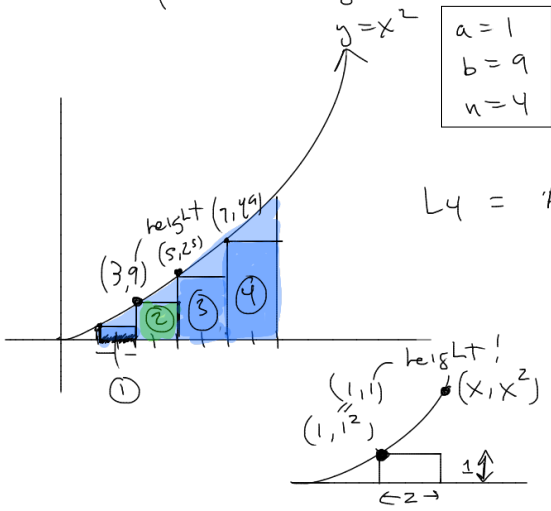
as $n \to \infty$

Approx gets better

Example

approximate

$f(x) = x^2$, find the area under curve from $x = 1$ to 9 using 4 rectangles



$a = 1$
$b = 9$
$n = 4$

width of rectangle

$\Delta x = \frac{b-a}{n} = \frac{9-1}{4} = \frac{8}{4} = 2$

L_4 = Area ① + Area ② + Area ③ + Area (4) = 168 units²

$\Delta x \cdot 1^2$	$\Delta x \cdot 3^2$	$\Delta x \cdot 5^2$	$\Delta x \cdot 7^2$
$2 \cdot 1$	$2 \cdot 9$	$2 \cdot 25$	$2 \cdot 49$
2	18	50	98

start @ 0 ↓ end @ 3 ↓

$= \Delta x f(x_0) + \Delta x f(x_1) + \Delta x f(x_2) + \Delta x f(x_3)$

$= \sum_{i=0}^{3} \Delta x f(x_i) = \Delta x \sum_{i=0}^{3} f(x_i)$ ← left endpts

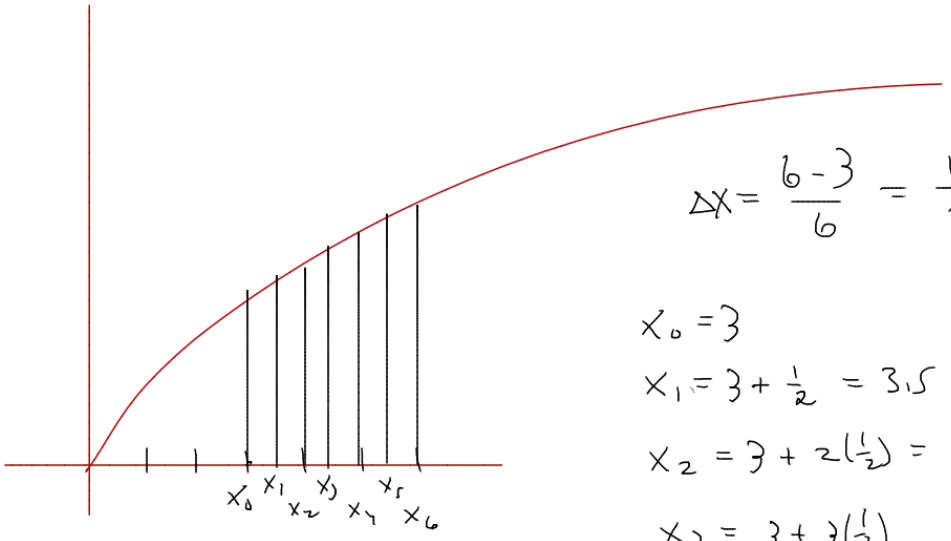
https://www.desqmos.com/calculator/tnawmju5t5

the precise area under the curve $f(x)$ from a to b

$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=0}^{n} f(x_i) \cdot \Delta x$$

Ex

$y = \sqrt{7x}$ on $[3;6]$ approx area under curve w/ M_6



$y = \sqrt{7x}$

$$\Delta x = \frac{6-3}{6} = \frac{1}{2}$$

$x_0 = 3$

$x_1 = 3 + \frac{1}{2} = 3.5$

$x_2 = 3 + 2(\frac{1}{2}) = 4.5$

$x_3 = 3 + 3(\frac{1}{2})$