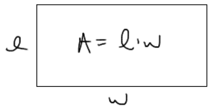
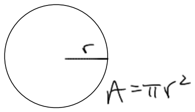
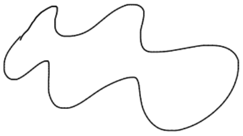
thur. wk 9

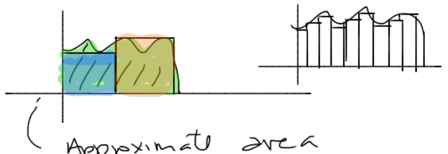
Ch. 5 · Riemann Sums & Area under Curve ⟶ Integral Calculus

Q: How to compute area









Approximate area w/ several smaller rectangles

More "smaller" rectangles give better approximation.

how the height of small rectangle is chosen

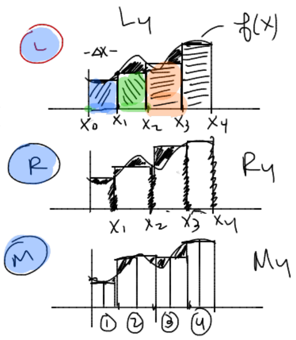
Methods:

left endpoint

right endpoint

mid point

typically better



4 sub-divisions
-> divide domain into 4 sub-intervals

Approximate Area w/ left endpoint method

$$L_4 = \overset{\text{Area Blue}}{\underbrace{\Delta x \cdot f(x_0)}_{\text{width × height}}} + \overset{\text{Area green}}{\Delta x \cdot f(x_1)} + \Delta x \cdot f(x_2) + \Delta x f(x_3)$$

$$= \Delta x \left(f(x_0) + f(x_1) + .. + f(x_3) \right) = \sum_{\underset{\text{start}}{n=0}}^{\overset{\text{stop}}{3}} \Delta x \cdot f(x_n)$$

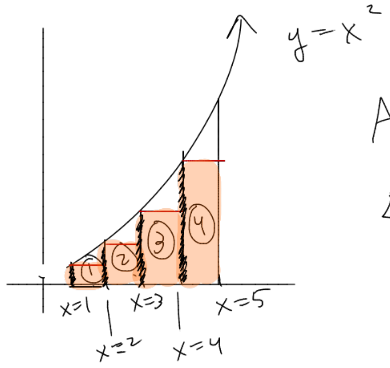
Note: Σ = "sum" & Δx being constant factors out

$$L_4 = \Delta x \cdot \sum_{n=0}^{3} f(x_n)$$

$$R_4 = \Delta x \sum_{n=1}^{4} f(x_n) \qquad M_4 = \Delta x \sum_{n=0}^{3} f\Big(\overset{\text{midpt formula}}{\frac{x_n + x_{n+1}}{2}}\Big)$$

Ex

$f(x) = x^2$, approximate area under graph on $[\overset{a}{1}, \overset{b}{5}]$ using $\underset{n}{4}$ rectangles w/ left endpts.



$$A \approx ① + ② + ③ + ④ = \underset{1}{\Delta x} \cdot \sum_{n=0}^{3} f(x_n)$$

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{4} = 1$$

$x_0 = 1$

$x_1 = 2$

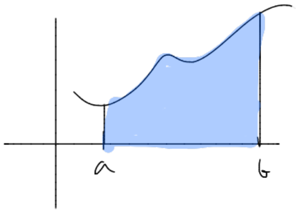
$x_2 = 3$

$x_3 = 4$

$$= 1 \cdot \left(\overset{1^2}{f(1)} + \overset{2^2}{f(2)} + \overset{3^2}{f(3)} + \overset{4^2}{f(4)} \right)$$

$$= 1 + 4 + 9 + 16 = 30$$

https://www.desmos.com/calculator/fnerrx7c0j the precise area under graph of f(x) is :



$$A_n(x) = \Delta x \cdot \sum_{i=0}^{n-1} f(x_i)$$

$$\lim_{n\to\infty} A_n(x) = \int_a^b f(x)\,dx$$ = area under curve from a to b.

$$\left(\text{as } n \to \infty,\ \underset{\text{actual width of rectangle}}{\Delta x} \to \overset{\text{infinitessimal change in x}}{dx} \right)$$