MA 161 - wk 9 - wed.

1. webwork dates extended thru Exam 3
2. Notes page: Exercises
   a. do them!

warm – up

2. $\int \frac{5-4x^2+3x^4}{x^3}\,dx = \int \frac{5}{x^3} - \frac{4x^2}{x^3} + \frac{3x^4}{x^3}\,dx = \int 5x^{-3} - 4x^{-1} + 3x\,dx$

power, $-4\cdot\frac{1}{x}$, power

$= \frac{5x^{-2}}{-2} - 4\ln|x| + \frac{3x^2}{2} + C$

Integrals Chart

$u = f(x)$	
$\int u^m \, du$	$\frac{u^{m+1}}{m+1} + C$
$\int e^u \, du$	$e^u + C$

kick it up by one
then divide by it

Ex.

$$\int (x^3+5)^{15} \cdot 4x^2\,dx$$

think: match chart $\int u^m du$ $\int e^u du$

what is u?
- often inside parenthesis
- often base of an exponent
- inside of some other function.

① $u = x^3 + 5$

② $\frac{du}{dx} = 3x^2$

③ $du = 3x^2\,dx$

④ $\frac{1}{3x^2}\,du = dx$

check.

$\frac{d}{dx}(\text{ans})$

$= \frac{4}{48} \cdot 16 (x^3+5)^{15} \cdot 3x^2$

$= 4(x^3+5)^{15} \cdot x^2$ ☺

use ①, ④ to sub

$$= \int (u)^{15} \cdot 4x^2 \frac{1}{3x^2}\,du = \int u^{15} \cdot \frac{4}{3}\,du = \frac{4}{3}\int u^{15}\,du = \frac{4}{3}\cdot\frac{u^{16}}{16} + C$$

back sub, get x back

$$= \boxed{\frac{4(x^3+5)^{16}}{48} + C}$$

Another power rule example:

Key for u-substitution: Identify derivative relationships, set u = to the mother.

$$\int \cos^5(x) \cdot \sin(x)\,dx = \int u^5 \cdot \sin(x) \cdot \left(\frac{-1}{\sin(x)}\right) du$$

$u = \cos(x)$

$\frac{du}{dx} = -\sin(x)$

$du = -\sin(x)\,dx$

$\frac{-1}{\sin x}\,du = dx$

$$= \int u^5 (-1)\,du = -\int u^5\,du = -\frac{u^6}{6} + C$$

$$= \boxed{-\frac{(\cos(x))^6}{6} + C}$$

check.

$\frac{d}{dx}(\text{ans}) = \frac{-6}{6}(\cos x)^5 \cdot (-\sin x)$

$= \cos^5 x \cdot \sin x$ ☺

Shortcut:

$$\frac{1}{5}\int (5x+1)^2 5\,dx = \frac{1}{5}\int u^2\,du = \frac{1}{15}u^3 + C = \frac{1}{15}(5x+1)^3 + C$$

$u = 5x + 1$

$\frac{du}{dx} = 5$

$du = 5\,dx$

think: match chart $\int u^n du$ $\int e^u du$

$$\int e^{x^3+1} \cdot x^2 \, dx$$

$u = x^3 + 1$ | $du = 3x^2 \, dx$

$\frac{du}{dx} = 3x^2$ | $\frac{1}{3x^2} du = dx$

$$\overset{\text{sub}}{=} \int e^u \cdot x^2 \cdot \frac{1}{3x^2} du = \int e^u \cdot \frac{1}{3} du = \frac{1}{3}\int e^u du = \frac{1}{3}e^u + C = \boxed{\frac{1}{3}e^{x^3+1} + C}$$

---

think: see only 1 e $\Rightarrow \int e^u du$

$$\int -\sin(x) e^{-(\cos x)} \, dx$$

$u = -\cos(x)$ | $\frac{1}{\sin x} du = dx$

$du = -(-\sin x) dx$

$du = \sin x \, dx$

$$\overset{\text{sub}}{=} \int -\sin x \, e^u \frac{1}{\sin x} du = -\int e^u du$$

$$= -e^{-\cos x} + C$$

check $\frac{d}{dx}(\text{ans}) = -e^{-\cos x} \cdot (-(-\sin x)) = -e^{-\cos x} \cdot \sin x$